RELIABILITY OF SYSTEMS EQUIPPED WITH VISCOUS DAMPERS WITH VARIABLE PROPERTIES

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Abstract. Viscous dampers are energy dissipation devices widely employed for the seismic control of structures. The performance of systems equipped with viscous dampers has been extensively analyzed by employing deterministic approaches. However, these approaches neglect the response dispersion due to the uncertainties in the input as well as in the structural system properties. Some recent works highlighted the important role of these uncertainties in the seismic performance of systems with linear or nonlinear viscous dampers. The present study focuses on the uncertainty in the damper properties and it aims at evaluating its influence on the probabilistic response of the damped system. In particular, the variability of the damper properties is assumed to be constrained by the tolerances allowed in qualification and production control tests. A preliminary study on the damper response is carried out to relate the constitutive damper characteristics to the parameters controlled in the experimental tests and to evaluate the consequences of damper parameter variations on the dissipation properties of the device. In the subsequent part of the study, the response hazard curves, providing the relation between the values of the response parameters of interest and the relevant yearly exceedance probability, are evaluated. In the analyses, a simplified structural system is considered, and the Subset Simulation (SS) algorithm is employed together with the Markov Chain Monte Carlo method to achieve a good estimate of small probabilities of exceedance. A sensitivity analysis, considering the expected variations in the damper properties, is finally carried out by employing the Augmented SS method to study the influence of the device acceptance ranges on the hazard curves.
1 INTRODUCTION

Supplemental energy dissipation systems are widely employed for the seismic control of new and existing structures. In particular, viscous dampers provide an efficient tool to dissipate the seismic input energy into heat, by reducing both the displacement and force demand in the structures [1, 2]. To date, the performance of systems equipped with viscous dampers has been extensively analyzed by employing deterministic approaches neglecting the response dispersion due to the uncertainties in the input as well as in the structural system properties.

However, these deterministic approaches provide only an approximate assessment of the seismic performance [3]. Some recent works have highlighted the important role of the uncertainties in the seismic performance of structures equipped with viscous dampers [4, 5, 6, 7] and the different propagation of ground motion uncertainties in systems equipped with linear or nonlinear viscous dampers [8, 9, 10]. The present study focuses on the influence of the uncertainties in the damper properties. In the approach suggested by codes and followed in practical design [11, 12, 13, 14], this uncertainty is bounded by the tolerances allowed in the qualification and production control tests. Consequently, design procedures usually involve a safety check based on the responses obtained by upper and lower bounds of the damper properties, chosen coherently with the test tolerances. However, the actual level of safety provided by the suggested design procedures is a problem requiring further investigation, as pointed out in [10], and the relevance of this topic is mainly due to the low robustness inherent to the structure-damper system, where the unexpected dissipative device failure can lead to a progressive collapse of the potentially non-ductile structure.

The present paper aims at evaluating the influence of these allowed tolerances in the probabilistic performance of the system, by providing useful information on the exceedance probability of the response parameters of most interest for the performance assessment (as described by response hazard curves) and by using an approach to the problem that is more efficient and reliable with respect to those used in previous studies on the same topic.

A preliminary analysis of the damper response is developed to relate the damper constitutive characteristics to the parameters controlled in the experimental tests and to analyze the variation in the dissipation properties of the device.

In the subsequent part of the study, response hazard curves are developed by employing a simplified model of the structural system. These curves provide the relation between the values of the response parameters of interest and the relevant yearly probability of exceedance. A sensitivity analysis, considering the expected variations in the damper properties, is then carried out to assess the influence of the device acceptance ranges on the hazard curves. The response measures considered include the maximum values of the deformation, related to the damage of the structural system and the damper failure [15], and the maximum values of the relative velocity, related to the damper force. The model uncertainties considered in the applications concern the earthquake scenario parameters and the ground motion characteristics, while other uncertainties concerning the structure response are neglected [16, 17]. The numerical applications, involving both linear and nonlinear viscous dampers show that tolerances allowed in the tests provide notable differences among the observed values of the response parameters of interest.

It is noteworthy that the probabilistic response of the system was analyzed in the previous studies by employing approaches consistent with the PEER framework [18]. This latter is a widely employed framework that permits a separation of the tasks related to the seismic hazard, structural vulnerability and expected losses assessment. The application of the framework is usually based on a description of the seismic input in terms of a small set of real records.
This approach does not permit to achieve an accurate estimate of small failure probabilities. For this reasons, in this study the Subset Simulation algorithm with Markov Chain Monte Carlo method are employed to obtain a good estimate of small probabilities of exceedance [19]. Coherently with this approach, the Augmented SS method is used for the sensitivity analysis [20]. This simulation techniques require a seismological stochastic model and the one proposed in [21, 22] has been used for the analyses.

2 DAMPER RESPONSE SENSITIVITY

The response of viscous damper is usually described by an exponential constitutive law in the form [23, 24]

\[ F_d = c|v|^\alpha \text{sgn}(v) \]  

(1)

where \( F_d \) is the measured force and \( c \) and \( \alpha \) are two constitutive parameters: the former is a multiplicative factor while the latter describes the nonlinear behaviour (\( \alpha = 1 \) for the linear case).

In a sensitivity study aiming at evaluating the consequences of variations of the damper parameters on the system performance, these two parameters could be assumed to vary freely.

However, the experimental procedure for the production controls suggest a different approach to the sensitivity analysis, involving characteristic parameters directly linked to the experimental test results, as explained hereafter. In general, the design is based on a target value of the maximum velocity \( v_0 \) attained at a circular frequency \( \omega_{0, \text{test}} \) relevant to the seismic response, and the production control tests are oriented to check the damper behavior at this design condition. More precisely, sinusoidal cycles simulating the design conditions, i.e., the displacement histories \( u(t) = v_0 / \omega_{0, \text{test}} \sin(\omega_{0, \text{test}} t) \), are imposed to the damper and the corresponding maximum damper force \( F_{d, \text{test}} \) is measured. Some tolerance is allowed in the force value and acceptance criteria usually requires that the difference between the measured value of the maximum force \( F_{d, \text{test}} \) and the expected (design) value \( F_{d0, \text{test}} \) is lower than +/- \( p F_{d0, \text{test}} \) [11, 12, 13]. The safety check is coherently carried out by considering the worst conditions compatible with the acceptance criteria [11, 12, 13], by adopting a lower/upper bound approach.

In this context, rather than investigating the system response by considering a free variation of the constitutive parameters it is more useful to link the variability of the response to the outcomes of the acceptance tests. Thus, in this study the response is investigated by assuming \( p \), describing the acceptance tolerance \( p F_{d0, \text{test}} \) as a system parameter and by introducing a second parameter \( \lambda \) to identify the different pairs \((c, \alpha)\) providing the same force variation \( p F_{d0, \text{test}} \). The pairs \((c, \alpha)\) satisfying the equality constraint lie on a curve, whose parametric expression is made explicit. Let \( c_0 \) and \( \alpha_0 \) denote the reference values assumed in the structural design. A variation \((\hat{c}, \hat{\alpha})\) of the constitutive parameters provides the following variation \( \hat{F}_{d0, \text{test}} \) of the maximum force \( F_{d0, \text{test}} \) expected in the sinusoidal test

\[ \hat{F}_{d0, \text{test}} = F_{d, \text{test}} - F_{d0, \text{test}} = F_{d0, \text{test}} \left[ 1 + \frac{\hat{c}}{c_0} \cdot (v_0)^{\hat{\alpha}} - 1 \right] \]  

(2)

The equation clearly shows that both \( \hat{c} \) and \( \hat{\alpha} \) contribute to this variation and the previous expression can be rewritten by introducing the tolerance parameter \( p \) as
\[ p = \left(1 + \frac{\hat{c}}{c_0}\right)(v_0)^{\hat{\alpha}} - 1 \]  \hspace{1cm} (3)

It is evident (Figure 1) that the same variation \( p \) can be obtained by different pairs \((\hat{c}, \hat{\alpha})\) and the constraint between these parameters can be described by introducing the parameter \( \lambda \):

\[ \hat{c}(p, \lambda) = c_0 \left(1 + \frac{p}{v_0}\right)^{\hat{\alpha}} - 1 \]  \hspace{1cm} (4)

\[ \hat{\alpha}(p, \lambda) = \lambda \]  \hspace{1cm} (5)

so that the pair \((p, \lambda)\) provides a representation of the response variability that is alternative to the representation provided by \((\hat{c}, \hat{\alpha})\). The condition \( \lambda = 0 \) coincides with a force variation due only to a variation of the response scale factor \( \hat{c} = p \), whereas the other values describe a force variation \( p \) involving a combination of \( \hat{c} \) and \( \hat{\alpha} \).

To complete the preliminary analysis of the damper variability effects, a linearized form of the previous relationship between the constitutive parameters is derived. This form can be of interest in the sensitivity analysis of problems where eqn.(1) is the only source of nonlinear behaviour and the local response variation can be conveniently approximated by analytical linear operators [25, 26, 27]. The relationships corresponding to eqns(3-5) are

\[ p = \frac{\hat{c}}{c_0} + \ln(v_0)\hat{\alpha} \]  \hspace{1cm} (6)

\[ \hat{c} = c_0[p - \lambda \ln(v_0)] \]  \hspace{1cm} (7)

\[ \hat{\alpha} = \lambda \]  \hspace{1cm} (8)

Figure 2 shows the parametric curves containing the pairs \((\hat{c}, \hat{\alpha})\), evaluated for 3 values of the force variation factor \( p \). The 3 cases reported correspond to \( p = -0.3, p = 0.0, p = +0.3 \).

These curves are obtained by intersecting the surfaces provided by eqn.(3) (nonlinear expression) and by eqn.(6) (linear approximation) with horizontal planes identifying the force variations. The results concerning the case of linear viscous damper \((\alpha = 1)\) and nonlinear viscous damper with \( \alpha = 0.2 \) are reported respectively in Figure 2a and Figure 2b. It can be ob-
served that the differences between the results obtained by using the nonlinear (exact) and the linear approximation become negligible in the case of the nonlinear damper. In this latter case, a notable percentage variation of \( \alpha \) is linked to very small variation of \( c \).

The energy dissipation properties are analyzed and reported in Figure 3. The energy dissipated in a cycle at the design conditions is denoted by \( W_{d0} \). The corresponding energy dissipated in the same test by a damper with different properties depend on \( p \) and \( \lambda \) and it is denoted by \( W_d \). Figure 3 shows the values of the ratio \( W_d / W_{d0} \) obtained by varying the parameters. The solid lines correspond to the results measured for fixed values of the force factor \( p \). The cases \( p = -0.30,-0.15,0,0.15,0.30 \) are reported and the points of these curves describe the variations relevant to different pairs \((\hat{c}, \hat{\alpha})\). Dashed lines connect the points with the same variation of \( c \). Results of Figure 3a refer to a linear viscous damper and Figure 3b refers to a nonlinear viscous damper with \( \alpha = 0.2 \). Given a value of the force factor \( p \), the energy dissipated changes significantly by varying the pair \((\hat{c}, \hat{\alpha})\). Higher values of \( \lambda = \alpha \) result in a reduction of the dissipative properties and this trend is more evident in the linear case. The discussed trend of the dissipation properties are valid in the neighborhood of the design conditions and different trends are observed for cycles with larger (or smaller) amplitudes which can be representative of the response for seismic inputs with intensity levels higher (or lower) than the design one.

Figure 2: Linear/nonlinear \( p-\alpha-c_d \) relationships for different \( p \) values \((-30; 0; +30 \%)\): a) \( \alpha_0 = 1.0 \); b) \( \alpha_0 = 0.2 \).

Figure 3. Dissipated energy variation in cyclic paths (amplitude 0.0454m, circular frequency \(2\pi\))
3 SEISMIC RESPONSE SENSITIVITY

3.1 Seismic performance

The seismic assessment and design of structures is usually carried out by evaluating via structural analysis the statistics of one (or more) response parameter of interest \( D \) and by ensuring that the \( P[D > d^*] \) of exceedance of a fixed threshold \( d^* \) is lower than an acceptable value \( P^* \). The random variable \( D \) can be expressed in terms of the ratio between a performance demand and the relevant capacity, so that a unitary threshold implies failure at ultimate condition. This way, \( D \) accounts for the uncertainties in the loading actions, in the model and the capacity. Typical response parameters employed in the context of the performance assessment of buildings with viscous dampers are: inter-storey drifts, absolute accelerations at storey, total base shear, forces on dampers, strokes on dampers.

Code design and assessment procedures aim at satisfying the reliability constraint in an indirect way, as they evaluate a conventional demand measure \( d_0 < d^* \) which is obtained from loadings of intensity whose exceedance probability \( P_0 \) is lower than the acceptable probability \( P^* \) [3]. Amplification factors are usually considered to pass from \( d_0 \) to \( d^* \).

In the specific case of the seismic analysis of buildings equipped with dampers, the design is carried out by evaluating the response for seismic events with a mean annual frequency of occurrence varying from \( 1 \times 10^{-2} \) (service limit state) to \( 2 \times 10^{-3} \) (ultimate limit state) [11, 28, 29] while the safety target requires that the probability of failure is lower than \( 10^{-5} - 10^{-6} \) [11, 29].

In order to relate the design condition, described by the pairs \( (d_0, P_0) \), to the effective system reliability, described by the couple \( (d^*, P^*) \), it is useful to evaluate the response hazard function

\[
G_d(d) = P[D > d] \tag{9}
\]

associating a generic value of the response parameter threshold \( d \) to the corresponding probability of exceedance. The conventional design value \( d_0 \) can be linked to the response value associated to a given probability of exceedance by introducing the inverse function \( G_d^{-1}(P) \) and defining the ratio

\[
\gamma(P) = \frac{G_d^{-1}(P)}{d_0} \tag{10}
\]

This ratio can be interpreted as the amplification factor for the design value \( d_0 \) providing the response parameter value corresponding to the desired probability of exceedance.

Design procedures generally do not involve a probabilistic analysis but aim at providing the value of \( d_0 \) by means of a deterministic analysis. For example, the seismic design is usually based on the description of the seismic input in terms of a pseudo-acceleration response spectrum and a reduced set of (artificial or natural) ground motions accounting for the record-to-record variability effects. The conventional design value of the response \( d_0 \) is obtained as the mean of the maximum response values. This design approach introduces further sources of approximation in the evaluation of the system reliability which are not addressed in this work.

3.2 Response hazard curve evaluation

Let \( X \) be the vector of the random variables of the system lying in the domain \( \Omega \), including both the variables describing the ground motion, for which a stochastic model is required,
and the structural system uncertainties. The system parameters are described by the vector \( \mathcal{G} \in \Gamma \), collecting the damper parameters \( p \) and \( \lambda \), as discussed in the previous section.

The failure corresponds to the region of response events such that \( G_d(d|\mathcal{G}) = \{x : g_d(x|\mathcal{G}) > d\} \), having denoted with \( g_d(x|\mathcal{G}) \) the response function providing the value of the parameter \( d \), once the sample \( x \) and the parameters \( \mathcal{G} \) are assigned. The response hazard function can be obtained as

\[
G_d(d|\mathcal{G}) = \int_{\Omega_G} I_d(x|\mathcal{G}) p_x(x) dx
\]

where \( p_x(x) \) if the probability density function (PDF) of the system variables and \( I_d(x|\mathcal{G}) \) is the indicator function, such that \( I_d = 1 \) if \( x \in G_d(d|\mathcal{G}) \) (equivalently \( g_d(x|\mathcal{G}) > d \)), otherwise \( I_d = 0 \). In the following, it is assumed that the \( p_x(x) \) is not influenced by the parameters \( \mathcal{G} \), although the formulation can be extended to the general case \( p_x(x|\mathcal{G}) \) [20].

The sensitivity problem is approached by considering the augmented reliability problem proposed in [20], i.e., by considering \( \mathcal{G} \) as a fictitious random variable with arbitrary PDF \( p_{\mathcal{G}}(\mathcal{G}) \). By this approach, the response sensitivity can be estimated with the same simulations employed for estimating the reliability, thus significantly reducing the computational effort with respect to other approaches.

The response hazard function can be obtained through the Bayes’ Theorem in the form

\[
G_d(d|\mathcal{G}) = \frac{p_{\mathcal{G}}(\mathcal{G}|d)}{p_{\mathcal{G}}(\mathcal{G})} G_d(d)
\]

where

\[
G_d(d) = \int_{\Omega_x} I_d(x|\mathcal{G}) p_x(x) p_{\mathcal{G}}(\mathcal{G}) dx d\mathcal{G}
\]

and

\[
p_{\mathcal{G}}(\mathcal{G}|d) = \frac{\int_{\Omega_x} I_d(x|\mathcal{G}) p_x(x) p_{\mathcal{G}}(\mathcal{G}) dx}{G_d(d)}
\]

In this formulation, both \( X \) and \( \mathcal{G} \) are random variables and the probability of exceeding the threshold \( d \) is a rare event that can be efficiently evaluated by the Subset Simulation-Markov chain method [19].

3.3 Uncertainties description

The seismic event is described by defining a seismic source characterized in terms of moment magnitude \( M \) and source-to-site (hypocentral) distance \( R \). The description of the uncertainty associated with the seismic input is completed by the specification of a stochastic ground motion model, considering the properties of the construction site. The intensity of the seismic event is described by the moment magnitude \( M \) and its uncertainty is modeled by the Gutenberg-Richter law defined on the interval \( [m_{\text{min}}, M_{\text{MAX}}] \) and corresponding to the following PDF of \( M \) given an earthquake event [30]:

\[
p_M(m) = \frac{\beta e^{-\beta m}}{\beta e^{-\beta m_{\text{min}}} - \beta e^{-\beta m_{\text{max}}}} \quad m \in [m_{\text{min}}, M_{\text{MAX}}]
\]
where \( \beta = \ln(10) b \) is a parameter related to the number of the expected earthquakes per annum with magnitude exceeding \( m \). More precisely, it is assumed that the occurrence of an event with \( M > m \) is a Poisson process with exceedance frequency \( \lambda(m) = 10^{a-bm} \) and no event is expected for \( M > m_{\text{MAX}} \). It is also assumed that no significant response is observed while its Fo \( U_{\text{BM}}, U_{\text{BM}} \), so the response hazard function, referred to a performance indicator. The two systems are characterized by very close values of \( u_0 \) for the probability of exceedance \( P_0 = G_{f}(u_0) = 0.0021 \) (10\% within 50 years); this probability value is usually assumed for the seismic design at ultimate limit conditions [11, 28, 29].
Table 1 reports the viscous damper properties ($\alpha_0$ and $c_0$) and the performances for the linear and nonlinear damped system at the design conditions. The response parameters considered for the performance comparison are: the maximum relative displacement $U$ (controlling the construction damage and the damper failure), the maximum relative velocity $V$ and the maximum force on the damper $F_d$ (controlling the damper failure), and the maximum absolute acceleration $A$ (controlling the damage of acceleration sensitive facilities).

<table>
<thead>
<tr>
<th>Viscous Properties</th>
<th>$\alpha_0$</th>
<th>$c_0$/m [g-s/m^2]</th>
<th>1.00</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performances</td>
<td>$u_0$ [m]</td>
<td>0.037</td>
<td>0.037</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$v_0$ [m/s]</td>
<td>0.276</td>
<td>0.285</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$F_{d0}$ [N]</td>
<td>208</td>
<td>146</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$a_0$ [m/s^2]</td>
<td>1.799</td>
<td>1.896</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Properties of the damping systems and relevant response at the design conditions.

Subset simulation analyses are performed to estimate the reliability of the damped systems for both the linear and nonlinear viscous dampers case. The first part of the next section compares the reliabilities of the reference systems (corresponding to the nominal damper properties), whereas the second part shows the effects of the variability of the damper viscous properties on the reliability. The probabilistic description of the system response is given in terms of hazard curves (complementary cumulative distribution function CCDF), providing the annual probability of exceedance for each relevant response parameters.

4.1 Probabilistic response at the reference conditions

In evaluating the seismic reliability of the systems at the reference conditions ($p = 0, \lambda = 0$), the set of uncertain parameters consists of those characterizing the earthquake excitation, i.e., magnitude, white-noise components and the model parameter $\varepsilon_{mod}$. As stated before (in Section 3.0), structural system parameters are assumed to be deterministic.

In order to estimate exceedance probabilities up to $10^{-6}$, subset simulations are carried out using 6 conditional levels (with threshold levels identified by a percentile equal to 10%) with 600 samples/level and a total number of samples equal to $540 \times 5 + 600 = 3300$. The averaged result of 10 independent simulations are presented.

Because of the adopted design criteria, the linearly and nonlinearly-damped systems show the same displacement $u_0$ at the probability of exceedance $P_0$ while the other response parameters exhibit different values. The ratios between the demand parameters and the reference values corresponding to the design conditions are reported in the Figure 4, so the unit values are located at the probability of exceedance $P_0$ and the values reported in the horizontal axis coincide with the ratio introduced in eqn.10. The maximum response parameters corresponding to the probability of exceedance $10^{-6}$ are highlighted in the figure by the coloured dashed lines, and relevant numerical values are collected in Table 2.

The linear and nonlinear damped systems exhibit different probabilistic responses within a range of exceedance probabilities up to $10^{-6}$. In fact, the exceedance probabilities of the maximum displacements and velocities are higher for the linear system than for the linear system, for values below the design value, and lower for values higher than the design value. An opposite trend is observed for the damper force, with the linear system requiring an higher strength capacity than the nonlinear one for rare seismic events. The accelerations show also a
different trend and the performance required by the nonlinear system are generally higher than those required by the linear system both for frequent and rare events. Qualitative trends observed are in agreement with results presented in previous studies obtained by conditional based approaches (PEER framework) [8, 10].

![Normalized response hazard curves](image)

Figure 4. Normalized response hazard curves for linear ($\alpha = 1.0$) and nonlinear ($\alpha = 0.2$) dampers.

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>1.00</th>
<th>0.20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_U(P = 10^{-6})$</td>
<td>7.20</td>
<td>14.32</td>
</tr>
<tr>
<td>$\gamma_{Fd}(P = 10^{-6})$</td>
<td>6.93</td>
<td>1.65</td>
</tr>
<tr>
<td>$\gamma_V(P = 10^{-6})$</td>
<td>6.64</td>
<td>12.54</td>
</tr>
<tr>
<td>$\gamma_A(P = 10^{-6})$</td>
<td>6.86</td>
<td>11.39</td>
</tr>
</tbody>
</table>

Table 2: Numerical values of $\gamma_d$ at $P = 10^{-6}$.

4.2 Effect of viscous damper properties variability

In this section, the influence of the variability of the viscous properties on the hazard curves is analyzed. The reference response evaluated in the previous section is compared with
the response obtained with the upper and lower bound of variations allowed in control production tests, as discussed in Section 2. The two limit values \( p = \pm 0.15 \) are assumed for the force \( \hat{F}_d|_{0,\text{test}} \) measured in the test at the design conditions, according to the limits suggested in [13]. Once \( p \) is assigned, only the parameter \( \lambda \) varies and gives the possible pairs of constitutive parameter variations \( \hat{\alpha} \) and \( \hat{c} \) (see eqns. 4-5).

The system response sensitivity is carried out by considering an “augmented reliability problem” [20] in which the \( \lambda \)-variability is added to the seismic uncertainties previously introduced. In order to investigate a realistic field of variation in the damper response, a uniform distribution function on \([-0.20, +0.20]\) is assumed for the system parameter, \( \lambda \), in this case coinciding with the nonlinear exponent variation \( \hat{\alpha} \). With the aim of increase the accuracy in the estimation of the exceedance probabilities a number of conditional samples greater than the number used in the previous section and equal to 1500 is used for each simulation level, for a total amount of samples equal to \( 1350 \times 5 + 1500 = 8250 \). The range of the parameter variation has been discretized in 4 bins \( J_i \) (i=1,...,4), and the hazard curves of the two extreme bins, centered at \( \hat{\lambda} = \pm 0.15 \) are reported in the following. An average of 50 independent simulations is used.

In Figures 5 the hazard curves of displacements, damper forces and absolute accelerations are plotted with a semi-logarithmic representation. Both the linear (left charts) and nonlinear (right charts) damped systems are considered, and the response parameters values are normalized by dividing them by the design value obtained in the reference design condition, as in the previous section. Each chart shows the reference curve (black solid line), a pair of blue dashed curves related to the upper bound \( p = +0.15 \) and describing the response variation for the two extreme bins of the parameter variation \( \hat{\lambda} = \pm 0.15 \), a pair of red dashed curve related to the lower bound \( p = -0.15 \) and describing the response variation for the two extreme bins of the parameter variation \( \hat{\lambda} = \pm 0.15 \).

In the linear case, the perturbed condition with \( p=-0.15 \) produces (moving towards the small probability range) an amplification on both the displacement and acceleration responses, while the opposite trend is observed for the damper force. The highest displacement variations occur when \( p=-0.15 \) and \( \hat{\lambda} \) is in the bin centred at the value \( \lambda = -0.15 \). This is a consequence of both the reduction of the damper force due to the negative value of \( p \) and the nonlinearity introduced by the negative variation of \( \hat{\alpha} \) (\( \alpha<1 \)) in the system behaviour, which results in larger displacements for less probable events (see Figure 4 for \( \hat{u} \) parameters). A different trend is observed for the forces, where the worst condition is represented by \( p=+0.15 \).

At the minimum probability of exceedance considered, the increment of the maximum displacement threshold is about 35% and the force threshold is about 20% while lower increments are observed for the absolute acceleration threshold.

Similar trends are shown in the nonlinear case where both the displacement and the acceleration demands grow for \( p=-0.15 \), while the damper forces increase when \( p=+0.15 \). However, in this case results are less sensitive to the parameter \( \hat{\lambda} \), expressing the percentile variation of the nonlinear exponent \( \alpha \).
Figure 5. Effects on the response hazard curves due to the perturbation on the damper parameters.
5 CONCLUSIONS

The paper analyzes the influence of both the seismic uncertainties and the variability of the viscous damper behaviour on the probabilistic response of passively protected systems. The probabilistic response is evaluated by means of advanced statistical simulation methods (Subset simulation with Markov chains), able to furnish an accurate estimation of the demand hazard for low values of the exceedance probability.

The effects of the variability of the viscous damper properties coherent with the tolerance range allowed by the codes for device control tests is studied via reliability-sensitivity analysis. This analysis is performed by carrying Subset simulations on an “augmented reliability problem”.

A comparison between the performances of two systems consisting of the same structure and of added linear ($\alpha=1$) and nonlinear ($\alpha=0.2$) viscous dampers is presented. The dampers are designed to achieve the same seismic performances (displacement response) at the usual design conditions suggested by codes of practice and corresponding to an annual probability of exceedance of 0.0021.

It is observed that the variability expected according to control production tests influences differently the response hazard curves for rare events. Considering probability of exceedance in the range $10^{-5}-10^{-6}$, it is observed that some response parameters are more sensitive to the damper parameter variations than others. Linear and nonlinear cases exhibit similar trends of variation but the amount of the response variations are very different in the two cases.

Current design procedures are based on the estimation of the probabilistic response and relevant structural safety by means of amplification factors increasing conventional design values of the structural response. The results observed in this study provide a contribution towards a more reliable definition of these amplification factors.

6 ACKNOWLEDGEMENT

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7 REFERENCES


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8 APPENDIX 1 - DETAILS OF ATKINSON-SILVA MODEL

The Atkinson-Silva ground motion model [21] used in this work is characterized by the radiation spectrum $A(f)$ and the time modulating function $e(t)$. The radiation spectrum gives a spectral representation of the ground motion at the construction site, accounting for several physical contributions influencing the wave propagation. Its analytical expression is

$$A(f) = \varepsilon_{mod} A_0(f) \cdot A_n(f) \cdot P_f(f) \cdot V(f)$$

The (two corner frequencies) point-source spectrum is represented by $A_0(f)$

$$A_0(f) = C \cdot M_0 \cdot (2\pi f)^2 \cdot \left(1 - \varepsilon\right) \cdot \frac{1}{1 + \left(f / f_a\right)^2} + \varepsilon \cdot \frac{1}{1 + \left(f / f_b\right)^2}$$

where, $M_0$ is the seismic moment (expressed in dyne-cm), related to the moment magnitude $M$ by
\[
M_0 = 10^{\frac{3}{2}(M+10.70)}
\]  
and C is a constant given by
\[
C = 10^{-20} \frac{\hat{R} \cdot V \cdot F_s}{4\pi \rho \beta^3}
\]
where \(\hat{R}, V, F_s\) are respectively the radiation pattern (\(\hat{R} = 0.55\)), a factor partitioning the total shear-wave energy into 2 horizontal components (\(V = 0.71\)) and the free-surface amplification factor (\(F_s = 2.0\)); \(\rho\) and \(\beta\) represent the soil density (\(\rho = 2.8t/m^3\)) and wave velocity (\(\beta = 3.5km/s\)) near the source; the multiplicative factor \(10^{-20}\) is in order to obtain \(cm\) as unit dimension for the ground motion (\(cm/s^2\) for accelerations). The two corner frequencies \(f_a\) and \(f_b\) and the \(\varepsilon\) parameter are related to the magnitude by
\[
\log(f_a) = 2.181 - 0.496 \cdot M
\]
\[
\log(f_b) = 1.380 - 0.227 \cdot M
\]
\[
\log(\varepsilon) = 3.223 - 0.670 \cdot M
\]
The \(A_n(f)\) function, characterizing the path effects of seismic waves, is given by
\[
A_n(f) = \frac{1}{R} e^{-\pi R f Q(f)}
\]
where the \(1/R\) term represents the geometrical spreading effect. The effect of the wave-transmission is accounted by the quality factor \(Q(f)\), defined as
\[
Q(f) = Q_0 f^n
\]
with \(Q_0 = 180\) and \(n = 0.45\) regional parameters. The \(P_f(f)\) function accounts for the path-independent loss of high-frequency in the ground motion and it is defined by
\[
P_f(f) = e^{(-\pi f)} \left[1 + \left(\frac{f}{f_{max}}\right)^8\right]^{-0.5}
\]
Whit \(k = 0.03\) and \(f_{max} = 100\) Hz. The soil amplification factor \(V(f)\) is taken according to [32] for generic soil (\(V_{S,30} = 310\) m/s). The model-error parameter \(\varepsilon_{mod}\) is the adding log-normal random variable (\(\mu_{inc} = 0, \sigma_{inc} = 0.5\)), according to Jalayer and Beck [31], used for increasing the record-to-record variability. For which concerns the envelope function \(e(t)\), it is given by
\[
e(t) = a \cdot \left(\frac{t}{T_n}\right)^b \exp\left(-c \cdot \left(\frac{t}{T_n}\right)\right)
\]
with parameters $b = -\varepsilon \cdot \frac{\ln(\eta)}{1 + \varepsilon(\ln(\varepsilon) - 1)}$, $c = \frac{b}{\varepsilon}$, $a = \left(\frac{\exp(1)}{\varepsilon}\right)^b$ and $\eta = 0.05$, $\varepsilon = 0.2$, as suggested in [22].

The ground-motion total duration is equal to $T_w = 2T_w$ with $T_w$ defined as

$$T_w = 0.05 \cdot R + \frac{1}{2f_a(M)}$$

(27)

The distance $R$ from the earthquake source to the site can be defined as follow, in function of the epicentral distance $r$ and the moment dependent nominal pseudo-depth $h$ ($\log(h) = 0.15M - 0.05$)

$$R = \sqrt{r^2 + h^2}$$

(28)