

# Multidisciplinary optimization by surrogate models: handling epistemic uncertainties by polynomial chaos expansion

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## ABSTRACT

We are interested in multidisciplinary optimization (MDO) of industrial structures such as aircraft. Performing MDO based on numerical high fidelity tools is challenging due to the cost of such solvers. Among all MDO architectures (see [1]), multidisciplinary feasible method (MDF) is known to be efficient and simple to implement. Unfortunately, its cost can be unaffordable as it requires a multidisciplinary analysis (MDA) of the system at each design evaluation. A way to tackle this issue is to replace each expert discipline solver by surrogate model [2, 3]. In addition to numerical cost reduction, this approach can be seen as an uncoupled architecture where each discipline is responsible for the training of its surrogate model (no MDA is performed for the computation of the surrogate). Nevertheless, let us assume that surrogate model evaluation returns the mean predicted value and its associated error approximation. This last information can be a probabilistic distribution (consider kriging surrogate model as an example) or some partial dispersion knowledge (see [2]). Finally, let us assume that outputs of surrogate models can be modeled by random variables (RV) whose distributions depend on the coupling variables.

As mentioned in [3], propagation of such model uncertainty in MDA context is challenging and only few studies have been proposed so far. To this respect, our main contribution is to carry out a functional representation of these model errors based on polynomial chaos expansion (PCE). As in usual application of PCE, each model error (ME) is expanded on a polynomial chaos basis with coefficients  $a^{ME,k,i}$  for  $i=0, \dots, P-1$  and  $k=1, \dots, n$  (where  $P$  is the size of the polynomial basis and  $n$  is the number of disciplines). But, as the probability distribution of the model error is a function of the coupling variables (which are RV), one has to consider that  $a^{ME,k,i}$  are also RV. We propose to expand these RV with respect to the independent RV of the underlying random space and to perform some fixed point iterations over the coefficients of the coupling variables PCE. As a result, a functional representation of the joint probability distribution of the coupling variables is obtained. From this functional representation, it is then straightforward to compute the PCE of the objective function for a given design. Moreover, PCE approach allows an analytical computation of Sobol' sensitivity indices [4]. These indices are used to determine which surrogate model must be improved to reduce the uncertainty on the objective function. At the end, the proposed methodology allows to quantify the uncertainty on the objective function due to the use of surrogate models. Then, the problem of finding a reliable or robust optimum is equivalent to MDO under aleatory uncertainties. The proposed method has been successfully tested on different test cases and will be experimented on more realistic ones. This work is supported by the european project H2020 AGILE.

## References

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