Performance of a domain decomposition method for high-order FEM

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ABSTRACT

Solving large scale Helmholtz problems is still an area of active research. The conventional finite elements method (FEM) leads to prohibitively expensive resolution requirements at high frequency. The high-order FEM, or pFEM, which provides improved efficiency and reduced computational costs is generally advocated.

The performance of the method relies heavily on efficient solving procedures for the resulting large, sparse, complex linear systems. On the one hand, continued efforts are deployed to develop efficient iterative solvers for this purpose. These methods scale well with the problem size, are highly parallelizable and generally require a limited amount of computational resources. However, due to the properties of the underlying operator, the design of robust iterative solvers for Helmholtz problems remains a challenge. On the other hand, direct solving procedures do not scale well with respect to problem size and frequency. In practice, the amount of computational resources available is often not sufficient to tackle the frequency range of interest.

An alternative to iterative or direct solving procedures is to resort to domain decomposition methods. A non-overlapping domain decomposition method called the Finite Element Tearing and Interconnecting method (FETI 2LM) employs Lagrange multipliers to recover the connections between the sub-domains. The method adopts an iterative procedure to solve an interface problem whereas a direct solver is used for the local problems. The performance of this method is well documented for conventional finite elements. In this paper, the focus is on the performance of the FETI 2LM for high-order FEM discretization. In addition to the dependency on the mesh size, frequency, or number of subdomains, the scalability of the method with respect to the order of approximation is assessed.

References

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