ECCOMAS Congress 2016
VII European Congress on Computational Methods in Applied Sciences and Engineering
M. Papadrakakis, V. Papadopoulos, G. Stefanou, V. Plevris (eds.)
Crete Island, Greece, 5–10 June 2016

COMPUTATIONAL METHOD OF DETERMINATION OF INTERNAL EFFORTS IN LINKS OF MECHANISMS AND ROBOT MANIPULATORS WITH STATICALLY DEFINABLE STRUCTURES CONSIDERING THE DISTRIBUTED DYNAMICALLY LOADINGS

Zh.Zh. Baigunchekov[1], M.U. Utenov[2], N.M. Utenov[3], S.K. Zhilkibayeva[4]

[1] Institute of Industrial Engineering, Kazakh National Research Technical University
Almaty, Republic of Kazakhstan
e-mail: bzh47@mail.ru

[2]Department of Mechanics, Kazakh National University
Almaty, Republic of Kazakhstan
e-mail: umu53@mail.ru

[3]Department of Mechanics, Kazakh National University
Almaty, Republic of Kazakhstan
e-mail: un.86@mail.ru

[4]Department of Mechanics, Kazakh National University
Almaty, Republic of Kazakhstan
e-mail: saltanatzhilkibayeva@gmail.com

Keywords: Manipulators, Mechanisms, Distributed Inertia Forces, Internal Loads, Kinematic Characteristic, Animation.

Abstract. *The technique of analytical determination of internal loads in links of planar rod mechanisms and manipulators with static definable structures taking into account the distributed dynamic stress, a self weight and the operating external loads is designed in this paper. The programs using the MAPLE are made on the given algorithm and animations of the motion of mechanisms with construction on links the intensity of cross and longitudinal distributed inertia loads, the bending moments, cross and longitudinal stress, depending on kinematic characteristics of links are obtained.*

1 INTRODUCTION

There are a variety of graph-analytical and numerical calculation methods on durability and rigidity of rod robotic systems and mechanisms, in which the distributed inertia forces of difficult character aren't considered [1-4]. The groups of Assur, that form the designed scheme of mechanism, can be statically definable, and also statically indefinable in concept of determination of internal stress. In this paper a new analytical approach of solution of problems of dynamic calculation on durability and rigidity taking into account the distributed dynamic stress in links of robotic systems and mechanisms with statically definable structures is proposed.

The distributed inertia forces of difficult character appear in links of rod mechanisms within the motion process. The intensity of distribution of inertia forces along the link depends on the mass distribution along the link and the kinematic characteristics of the mechanism changing rapidly. Rise of that sort of loads causes a set of problems, namely, breaking problems, which are specified by large-scale inertia forces; significance of elastic deformation of mechanism, that puts the mechanism out of action; because of deformation of links the mechanism can't meet kinematic claims.

Therefore, relations between the intensity of distributed inertia forces and a self weight of links with geometrical, physical and kinematic characteristics are determined in our work. The laws of distribution of inertia forces and self weight allow to output laws of distribution of internal forces on the axis of link in each position of links, where there is a force attached to any point of a link. Their maximum values allow to optimize the design data of a link, which provides durability and rigidity of links and, entirely, of robotic systems and mechanisms.

As internal loads of each continual link are defined unambiguously by a set of internal loads in its separate cross-sections and by the matrixes of approximations, so the task is to calculate the internal loads in finite number of cross-sections of elements.

As a result, we refer to discrete model of elastic calculation of links of rod mechanisms. For elastic calculation of rod mechanisms based on Dalamber's principle, mechanisms are casted to structures which degree of freedom is equal to zero. For definition of internal loads in links of designed scheme of mechanism, the structure is divided into elements, both the hinged and rigid joints. The elements are divided into three types of beams for the first time. Discrete models of these three types of the beams with constant cross-sections which are under the action of cross and the longitudinal distributed loads of a trapezoidal view are constructed. The constructed discrete models for these three types of beams with constant cross sections along the axis allow to determine quantity of the independent dynamic equations of balance, components of a vector of forces in calculated cross-sections and to construct discrete model of all structure.

The dynamic equations of balance for discrete model of an element of the link with constant cross-sections which is under the influence of cross and longitudinal inertial loads of a trapezoidal look are also received in this work as well as the equations of balance of hinged and rigid knots expressed through required parameters of internal forces.

If we unite the equations of dynamic balance of elements and knots in one system, we will receive the equations of dynamic balance of all discrete model of system. A sort of systems of equations is sufficient for definition of internal forces in links of mechanisms, which structure is a static definable. The vector of forces and vector of loads in calculated cross-sections of discrete models of mechanisms are formed from vectors of forces and vectors of loads in calculated cross-sections of their separate elements. On the given algorithm the programs in the MAPLE system are made and animations of the motion of mechanisms with construction on links the intensity of cross and longitudinal distributed inertia loads, the bending moments, cross and longitudinal forces, depending on kinematic characteristics of links are obtained.

2. INERTIA FORCES AND MATRIX OF APPROXIMATIONS

Considering the plane-parallel motion of an ***k***th link of mechanism with constant cross-sections comparatively fixed system of coordinates *OXY*, the following laws of distribution of the cross and longitudinal inertia forces along a link, that arise from self mass of a link are defined [5]:

$$q_k\left(x_k^{'}\right) = a_{kq} + b_{kq}x_k^{'}$$
$$n_k\left(x_k^{'}\right) = a_{kn} + b_{kn}x_k^{'} \qquad (1)$$

where $a_{kq} = -\gamma_k A_k \cos\theta_k - \frac{\gamma_k A_k}{g} w_{kp}^{y_k^{'}}$, $b_{kq} = -\frac{\gamma_k A_k}{g}\varepsilon_k$, $a_{kn} = -\gamma_k A_k \sin\theta_k - \frac{\gamma_k A_k}{g} w_{kp}^{x_k^{'}}$, $b_{kn} = \frac{\gamma_k A_k}{g}\omega_k^2$, θ_k - an angle, which determines the position of the ***k***th link comparatively fixed system of coordinates *OXY*, respectively, ω_k, ε_k - angular velocity and angular acceleration of the ***k***th link, respectively, $w_{kp}^{x_k^{'}}$ и $w_{kp}^{y_k^{'}}$ - components of P_k (pole) point acceleration of the ***k***th link put on the axis of link and perpendicular to it, respectively, γ_k - specific weight of material of the ***k***th link, A_k - square of cross-section of the ***k***th link, g - acceleration of gravity.

The obtained expressions show that the distribution of cross and longitudinal inertia forces along the axis of link with constant cross-sections is characterized by trapezoidal law.

For the ***k***th link, which is under the influence of longitudinal trapezoidal distributed stress, Fig. 1, the bending moments along the length of element are distributed by the law of polynomial of third-degree.

$$M_k\left(x_k^{'}\right) = a_0 + a_1 x_k^{'} + a_2\left(x_k^{'}\right)^2 + a_3\left(x_k^{'}\right) \qquad (2)$$

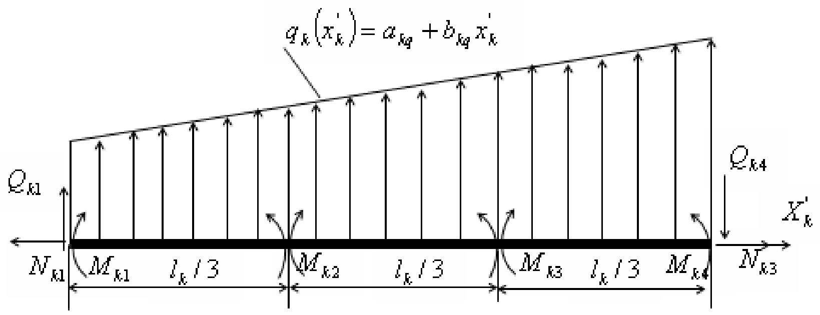

Figure 1: Longitudinal trapezoidal distributed load acting on the element

Now, let express the bending moments in $x_k^{'}$ cross-section through the sought bending moments M_{k1}, M_{k2}, M_{k3}, M_{k4} in the cross-sections demonstrated in Fig. 1, respectively. For this purpose it is enough to express coefficients a_0, a_1, a_2, a_3 through $M_{k1}, M_{k2}, M_{k3}, M_{k4}$. As a result we have [6]:

$$M_k\left(x_k^{'}\right)=\left[1-\frac{11}{2l_k}x_k^{'}+\frac{9}{l_k^2}\left(x_k^{'}\right)^2-\frac{9}{2l_k^3}\left(x_k^{'}\right)^3\right]M_{k1}+\left[\frac{9}{l_k}x_k^{'}-\frac{45}{2l_k^2}\left(x_k^{'}\right)^2+\frac{27}{2l_k^3}\left(x_k^{'}\right)^3\right]M$$
$$+\left[-\frac{9}{2l_k}x_k^{'}+\frac{18}{l_k^2}\left(x_k^{'}\right)^2-\frac{27}{2l_k^3}\left(x_k^{'}\right)^3\right]M_{k3}+\left[\frac{1}{l_k}x_k^{'}-\frac{9}{2l_k^2}\left(x_k^{'}\right)^2+\frac{9}{2l_k^3}\left(x_k^{'}\right)^3\right]M_{k4k2} \quad (3)$$

Differentiating $M_k\left(x_k^{'}\right)$ to $x_k^{'}$ gives the equation of shear force:

$$Q_k\left(x_k^{'}\right)=\left[-\frac{11}{2l_k}+\frac{18}{l_k^2}x_k^{'}-\frac{27}{2l_k^3}\left(x_k^{'}\right)^2\right]M_{k1}+\left[\frac{9}{l_k}-\frac{45}{l_k^2}x_k^{'}+\frac{81}{2l_k^3}\left(x_k^{'}\right)^2\right]M_{k2}+$$
$$\left[-\frac{9}{2l_k}+\frac{36}{l_k^2}x_k^{'}-\frac{81}{2l_k^3}\left(x_k^{'}\right)^2\right]M_{k3}+\left[\frac{1}{l_k}-\frac{9}{l_k^2}x_k^{'}+\frac{27}{2l_k^3}\left(x_k^{'}\right)^2\right]M_{k4}. \quad (4)$$

Let the element be affected by the longitudinal trapezoidal distributed load, except the distributed shear force. In that case, the longitudinal force in arbitrary cross-section of an element can be expressed analogously to previous by means of longitudinal forces in calculated cross-sections as follows:

$$N_k\left(x_k^{'}\right)=\left[1-\frac{3}{l_k}x_k^{'}+\frac{2}{l_k^2}\left(x_k^{'}\right)^2\right]N_{k1}+\left[\frac{4}{l_k}x_k^{'}-\frac{4}{l_k^2}\left(x_k^{'}\right)^2\right]N_{k2}+\left[-\frac{1}{l_k}x_k^{'}+\frac{2}{l_k^2}\left(x_k^{'}\right)^2\right]N_{k3} \quad (5)$$

Thus, for the element which is acted by cross and the longitudinal trapezoidal distributed loads, the approximation matrix connecting internal loads in arbitrary cross-section of the element with values of internal loads in cross-sections has an appearance:

$$\left[H_k\left(x_k^{'}\right)\right]=\begin{bmatrix} h_{11}\left(x_k^{'}\right) & h_{12}\left(x_k^{'}\right) & h_{13}\left(x_k^{'}\right) & h_{14}\left(x_k^{'}\right) & 0 & 0 & 0 \\ h_{21}\left(x_k^{'}\right) & h_{22}\left(x_k^{'}\right) & h_{23}\left(x_k^{'}\right) & h_{24}\left(x_k^{'}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_{35}\left(x_k^{'}\right) & h_{36}\left(x_k^{'}\right) & h_{37}\left(x_k^{'}\right) \end{bmatrix} \quad (6)$$

Elements of the first line of this matrix can be seen from the Eq. (3), elements of the second line can be seen from the Eq. (4), and elements of the third line can be seen from the Eq. (5), respectively.

The given expression of a matrix of approximation of loads defines dependence between a vector of forces $\left\{S_k\left(x_k^{'}\right)\right\}$ in arbitrary section of an $x_k^{'}$ element and a vector of forces $\left\{S_k\right\}$ in the appointed cross-sections. For an element of rod system the matrix of approximation is obtained accurately as it is solved on the basis of known laws of distribution of sought forces.

Note, the equations of the bending moment, the cross and longitudinal forces (3,4,5) respectively, which are expressed by the same values in calculated cross-sections, show that for definition of internal loads of each element of the mechanism it is enough to know values of these loads in final number of cross-sections of each of these elements. Number of sections in which it is necessary to know values of internal loads, are defined by polynomial degrees of external actions. Thus, internal loads of each continual link are determined unambiguously by a set of internal loads in its separate cross-sections and by the matrixes of approximations, therefore, the task is reduced to calculation of internal forces in final number of cross-sections of elements. Hence, we come to a discrete model of elastic calculation of links of rod mechanisms.

3. DISCRETE MODELS OF ELASTIC CALCULATION OF ELEMENTS AND MECHANISMS IN GENERAL

For elastic calculation of rod mechanisms based on Dalamber's principle, all inertial, external forces, gravity of links are attached and the unknown driving moments (forces) are applied to support assigned laws of their motion. If the ground hinges connecting drive link with rigid fixed-end are replaced, then the frames which joint is equal to zero are received.

For definition of internal stress in links (in elements) of calculated scheme of mechanism, the frame is divided into elements and joints. The link or its part can act as the elements, whereas the joints are ground hinges connecting links and cross-sections in the middle part, where concentrated external stress is occurred.

The process of frame sectioning consists of giving function and signs for element calculated section. While dividing the elements of calculated scheme of frame into calculated cross-sections and joints, it is necessary to set what internal relations between elements are remained or removed. If we reject any internal relations or their combinations in the element, so the element breaks up to two elements which can turn, move or be removed relatively each other. With the purpose to prevent it, internal forces-loads have to be applied at the joint rejecting places. Thereafter, these loads are regarded as primary unknowns.

Let's decompose an element of planar rod mechanisms on three types of beams, for convenience of working up the solving equations to determine the internal loads in the appointed cross-sections of elements of the mechanism [7].

Such beams can be the rods of basic linkage, if they are connected among themselves rigidly.

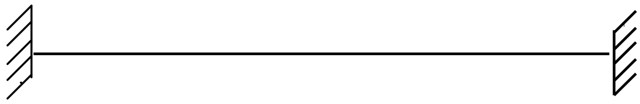

Figure 2: Beam's both ends are fixed rigidly (first type of a beam)

For determination of coefficients of expressions of the bending moment, it is necessary to know values of the bending moments in four cross-sections, and for determination of coefficients of expressions of longitudinal force, it is necessary to know values in three sections of an element. Therefore, we will choose four sections with unknown bending moments and three sections with unknown longitudinal forces in this beam. Then, by means of conditional schemes with the corresponding unknown, we will construct discrete model of the considered beam, Fig. 3.

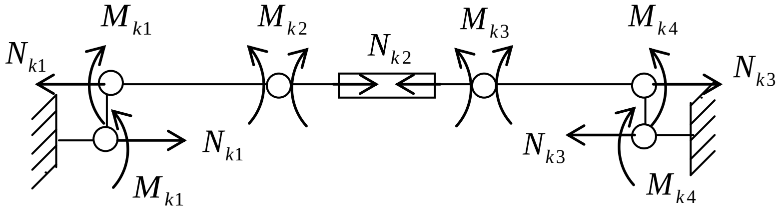

Figure 3: Discrete model of the first type beam under the action of the distributed trapezoidal load

Then the vector of forces in calculated cross-sections of the beam's discrete model is expressed by the following vector:

$$\{S_k\} = \{M_{k1}, M_{k2}, M_{k3}, M_{k4}, N_{k1}, N_{k2}, N_{k3}\}^T \quad (7)$$

There is dependence between degree of freedom of discrete model m, number of the attached external loads n and degree of redundancy of calculated scheme k [8]:

$$m = n - k \quad (8)$$

The matter is that total number of loads n of calculated cross-sections is counted easily, and degree of redundancy of calculated scheme is obtained by formula $k = 3K - Ш$, where K - number of the closed contours, $Ш$ - number of simple (single) joints, k - degree of redundancy of calculated scheme of mechanism.

Degree of freedom of discrete model m determines the quantity of necessary independent equations of statics.

Let's define the degree of freedom of discrete model of this beam. For this discrete model of beam the number of unknowns $n = 7$, the redundancy of beam $k = 3$, so the degree of freedom of discrete model $m = 4$. In other words, it is possible to work out four independent equilibrium equations for this discrete model of beam.

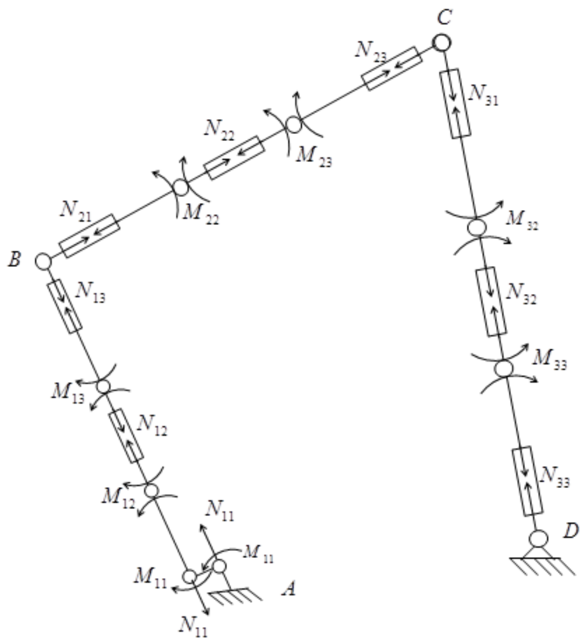

Figure 4: Discrete model of the four-link mechanism
with constant cross-sections of links

The second type of an element is this beam, which one end is fixed rigidly and other end – joint-fixed. As an example, it can be drive links of planar rod mechanisms. The elements of the third type are beams of interlinks. They can be considered as the beams joint-fixed on the ends. The discrete models for beams of the second and third type are constructed similarly to the first type of beams.

The discrete model of the four-bar mechanism is constructed on Fig. 4; all sought values are shown here, these help to define all internal forces in any cross-section of rods of the mechanism.

For the first link (the second type of a beam) of this mechanism the vector of forces in cross-sections $\{S_1\}$ has the following components:

$$\{S_1\}=\{M_{11},M_{12},M_{13},N_{11},N_{12},N_{13}\}^T \tag{9}$$

For the second and third links (the third beam type) regard to this mechanism, the vector of forces in calculated cross-sections have the following components, respectively:

$$\{S_2\}=\{M_{22},M_{23},N_{21},N_{22},N_{23}\}^T;\{S_3\}=\{M_{32},M_{33},N_{31},N_{32},N_{33}\}^T \tag{10}$$

For all discrete model of the mechanism the vector of forces in calculated cross-sections has an appearance:

$$\begin{gathered}\{S\}=\{\{S_1\},\{S_2\},\{S_3\}\}^T = \\ \{M_{11},M_{12},M_{13},N_{11},N_{12},N_{13},M_{22},M_{23},N_{21},N_{22},N_{23},M_{32},M_{33},N_{31},N_{32},N_{33}\}^T\end{gathered} \tag{11}$$

4. DYNAMIC EQUATIONS OF EQUILIBRIUM OF DISCRETE MODELS OF ELEMENTS AND JOINTS

Let's remove the equations of dynamic equilibrium of an element. From the attached concentrated external loads (Q_{k1},M_{k1}) and from the cross trapezoidal distributed loads on the axis of element, in arbitrary cross-section of $x_k^{'}$ element there is a bending moment determined by Eq. (2). On the other hand, the bending moment in cross-section of $x_k^{'}$ element, which is expressed through the sought moments in calculated cross-sections, is solved by Eq. (3).

If the Eq. (2) and Eq. (3) will be differentiated three times on $x_k^{'}$, then they will be equated and substituted to value b_{kq}, respectively, then the primary equation of dynamic equilibrium of element will be:

$$-\frac{27}{l_k^3}M_{k1}+\frac{81}{l_k^3}M_{k2}-\frac{81}{l_k^3}M_{k3}+\frac{27}{l_k^3}M_{k4}=-\frac{\gamma_k A_k}{g}\varepsilon \tag{12}$$

Relation between the values of the unknown quantities of bending moments in the calculated cross-sections and geometric, physical and kinematic characteristics of ***k***th element of mechanism is found. Thus, the second equation is expressed through relation of the sum of moments of all the acting forces on k - element to center of gravity of $k4$ cross-section, Fig. 1:

$$Q_{k1}l_k+a_{kq}\frac{l_k^2}{2}+b_{kq}\frac{l_k^3}{6}+M_{k1}-M_{k4}=0; \tag{13}$$

where $Q_{k1}=-\frac{11}{2l_k}M_{k1}+\frac{9}{l_k}M_{k2}-\frac{9}{2l_k}M_{k3}+\frac{1}{l_k}M_{k4}$, this equations is easy to get, if the value $x_k^{'}=0$ is substituted into the Eq. (4).

Substituting the values Q_{k1} and a_{kq}, b_{kq} into Eq. (10), respectively, and summing the coefficients of unknowns of the same name, and also known quantities in the right end of the equation, the second equilibrium equation can be written as:

$$-\frac{9}{2}M_{k1}+9M_{k2}-\frac{9}{2}M_{k3}=\left(\gamma_k A_k\cos\theta_k+\frac{\gamma_k A_k}{g}w_{k1}^{y_k^{'}}\right)\frac{l_k^2}{2}+\frac{\gamma_k A_k}{g}\varepsilon_k\frac{l_k^3}{6} \tag{14}$$

From the longitudinal trapezoidal distributed loads acting on the element, as well as from the force N_{k1} of the $k1$ cross-section, in the $x_k^{'}$ cross-section of element the longitudinal force is occurred, which can be solved by equation:

$$N_k\left(x_k^{'}\right)=N_{k1}-a_{kn}x_k^{'}-b_{kn}\frac{\left(x_k^{'}\right)^2}{2} \tag{15}$$

On the other hand, the longitudinal force in the $x_k^{'}$ cross-section of the element, expressed by means of longitudinal forces in the calculated cross-section, has the form (5).

Differentiating twice on $x_k^{'}$ the Eq. (12 and 5), respectively, equating them and substituting the value b_{kn}, the third equation of equilibrium can be expressed as:

$$\frac{4}{l_k^2}N_{k1}-\frac{8}{l_k^2}N_{k2}+\frac{4}{l_k^2}N_{k3}=-\frac{\gamma_k A_k}{g}\omega_k^2 \tag{16}$$

Projecting all forces acting on the ***k***th element on the $x_k^{'}$ axis and substituting the values a_{kn}, b_{kn} the third equation of equilibrium is found. Thus

$$-N_{k1}+N_{k3}=\left(\gamma_k A_k\sin\theta_k+\frac{\gamma_k A_k}{g}w_{k1}^{x_k^{'}}\right)l_k-\frac{\gamma_k A_k}{g}\omega_k^2\frac{l_k^2}{2} \tag{17}$$

Obtained system of equation, which consist of Eq. (9), (11), (13) and (21) are assembled in a matrix form as:

$$[A_k]\{S_k\}=\{F_k\} \tag{18}$$

where

$$[A_k]=\begin{bmatrix} -\frac{27}{l_k^3} & \frac{81}{l_k^3} & -\frac{81}{l_k^3} & \frac{27}{l_k^3} & 0 & 0 & 0 \\ -\frac{9}{2} & 9 & -\frac{9}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{4}{l_k^2} & -\frac{8}{l_k^2} & \frac{4}{l_k^2} \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

$$\{S_k\}=\{M_{k1},M_{k2},M_{k3},M_{k4},N_{k1},N_{k2},N_{k3}\}^T$$

$$\{F_k\}=\left\{b_{kq},\quad -a_{kq}\frac{l_k^2}{2}-b_{kq}\frac{l_k^3}{6},\quad -b_{kn},\quad -a_{kn}l_k-b_{kn}\frac{l_k^2}{2}\right\}^T$$

Then, in the section of the element adjacent to the site (for kinematic pair) there are internal forces, as shown in Figure 5. For these units have two equilibrium conditions:

Let the two elements j and k of mechanism form a rotational kinematic pair, i.e. permit rotational motion relative to each other. Also let the length of these elements has a constant cross-section. Cut out of the mechanism a kinematic pair with surrounding cross-sections of the elements constituting this pair. Then, in the cross-section of the element adjacent to the joint (to kinematic pair) there are internal loads, as shown in Fig. 5. There are two equilibrium conditions for these joints:

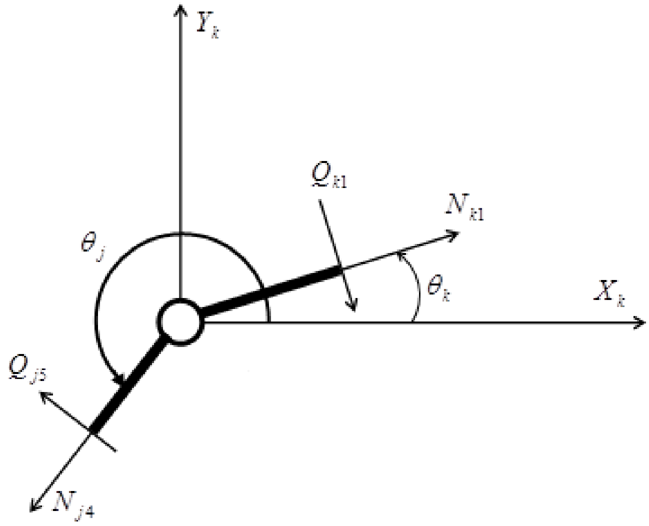

Figure 5: The hinge joint mechanism with constant cross-section elements

The equation of equilibrium for this joint can be described as:

$$\begin{cases} N_{k1}\cos\theta_k + Q_{k1}\sin\theta_k + N_{j3}\cos\theta_j + Q_{j4}\sin\theta_j = 0; \\ N_{k1}\sin\theta_k - Q_{k1}\cos\theta_k + N_{j3}\sin\theta_j - Q_{j4}\cos\theta_j = 0 \end{cases} \quad (19)$$

Further, the values will be expressed by means of sought moments in the calculated cross-sections of discrete model of the element, for this purpose we use the Eq. (4), substituting here the values $x_k^{'} = 0$ and $x_j^{'} = l_j$, respectively, hence:

$$Q_{k1} = -\frac{11}{2l_k}M_{k1} + \frac{9}{l_k}M_{k2} - \frac{9}{2l_k}M_{k3} + \frac{1}{l_k}M_{k4};$$
$$Q_{j4} = -\frac{1}{l_j}M_{j1} + \frac{9}{2l_j}M_{j2} - \frac{9}{l_j}M_{j3} + \frac{11}{2l_j}M_{j4}$$

Now, substituting the values Q_{k1} and Q_{j4} in the Eq. (19), the following equilibrium equations for joint have an appearance:

$$\begin{cases} -\frac{11\sin\theta_k}{2l_k}M_{k1} + \frac{9\sin\theta_k}{l_k}M_{k2} - \frac{9\sin\theta_k}{2l_k}M_{k3} + \frac{\sin\theta_k}{l_k}M_{k4} + \cos\theta_k N_{k1} - \\ -\frac{\sin\theta_j}{l_j}M_{j1} + \frac{9\sin\theta_j}{2l_j}M_{j2} - \frac{9\sin\theta_j}{l_j}M_{j3} + \frac{11\sin\theta_j}{2l_j}M_{j4} + \cos\theta_j N_{j3} = 0; \\ \frac{11\cos\theta_k}{2l_k}M_{k1} + \frac{9\cos\theta_k}{l_k}M_{k2} + \frac{9\cos\theta_k}{2l_k}M_{k3} - \frac{\cos\theta_k}{l_k}M_{k4} + \sin\theta_k N_{k1} + \\ + \frac{\cos\theta_j}{l_j}M_{j1} - \frac{9\cos\theta_j}{2l_j}M_{j2} + \frac{9\cos\theta_j}{l_j}M_{j3} - \frac{11\cos\theta_j}{2l_j}M_{j4} + \sin\theta_j N_{j3} = 0. \end{cases} \quad (20)$$

The linkage cross-sections can be as rigid joints, the concentrated external loads are attached here. For example, the concentrated forces $P_{kx'_k}$, $P_{ky'_k}$ and the concentrated moment M_k are occurred in the G cross-section of ***k***th link, Fig.5.

Then the k link is divided into two elements, for kth and ith. If the cross-sections of elements are constant along the length of the link, then by means of cutting the G joint out of mechanism, the scheme of G joint with adjacent internal loads in cross-sections is displayed below. For this joint it is possible to write the three equilibrium equations that are expressed through sought parameters of elements.

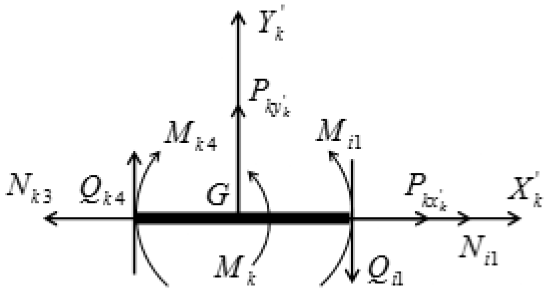

Figure 6: Rigid joints of a link with a constant cross-section of elements, where the external concentrated forces are attached

5. RESOLVING EQUATIONS OF DETERMINATION OF INTERNAL FORCES

By combining the equilibrium equations of elements and joints into a single system, the equilibrium equations of the discrete model of entire mechanism is obtained. They can be written in general form:

$$[A]\{S\}=\{F\} \tag{21}$$

Such systems of equations are sufficient to determine the internal forces in the links of the mechanism, which frame includes a statically definable group of Assur.

The matrix of equilibrium equations for the discrete model of mechanisms consists of matrices of equilibrium equations of their individual elements, as well as the equilibrium equations of their joints. The matrix of dynamic equilibrium equations of discrete models of mechanisms is as follows:

$$[A]=\begin{bmatrix} [A_1] & 0 & . & . & . & 0 \\ 0 & [A_2] & . & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & [A_n] \\ \multicolumn{6}{c}{the\ equilburium\ equations} \end{bmatrix}$$

The vector of force and the vector of load in calculated cross-sections for discrete models of mechanisms is formed by vector of forces and loads in calculated cross-sections of their separate elements. These vectors in vector form, respectively, have the following species:

$$\{F\}=\{\{F_1\},\{F_2\},...,\{F_n\}\}^T;\{S\}=\{\{S_1\},\{S_2\},...,\{S_n\}\}^T$$

Now, for determination of internal loads in links, we give an example of six-bar second class mechanism with single drive linkage as shown in Fig. 7. The computer programs for determination and construction of the inertia forces and internal loads on the links by means of using the MAPLE system are made. Therefore, the results of obtained inertia forces and internal loads for some positions of the mechanism are shown in Figs. 7-12.

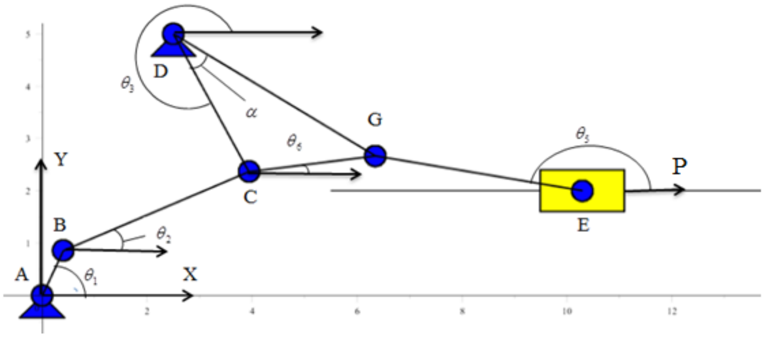

Figure 7: A six-bar second class mechanism with single drive linkage

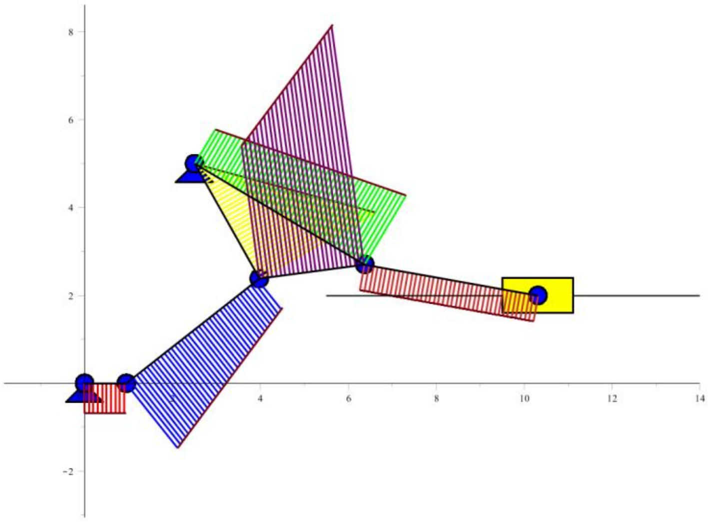

Figure 8: The investigating mechanism,
on which links the diagrams of cross inertia forces are constructed

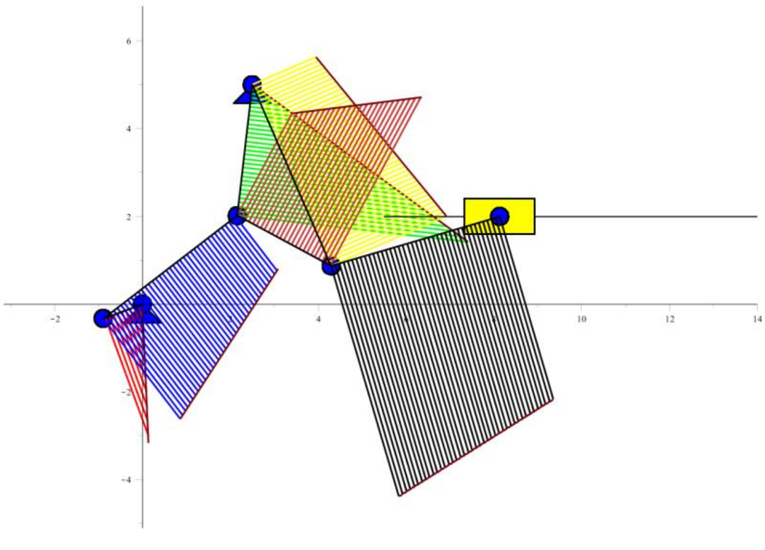

Figure 9: The investigating mechanism,
on which links the diagrams of longitudinal inertia forces are constructed

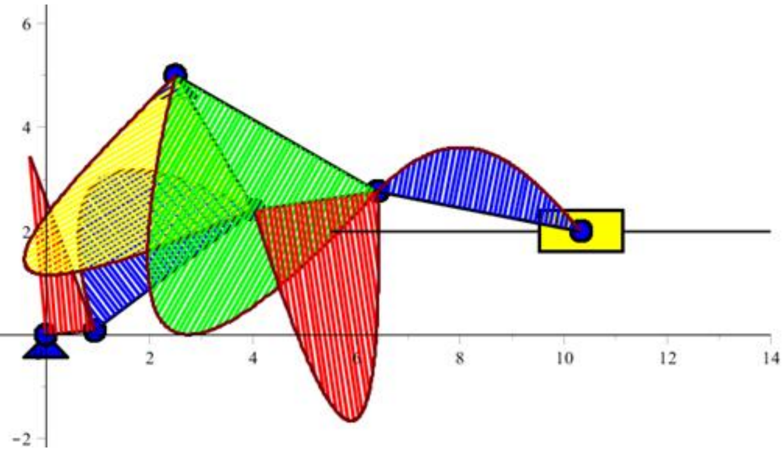

Figure 10: The investigating mechanism,
on which links the diagrams of bending moments are constructed

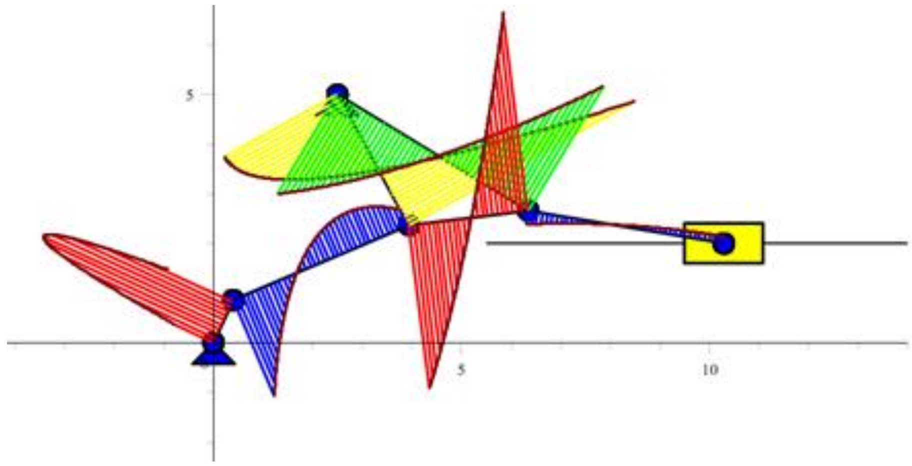

Figure 11: The investigating mechanism,
on which links the diagrams of shearing forces are constructed

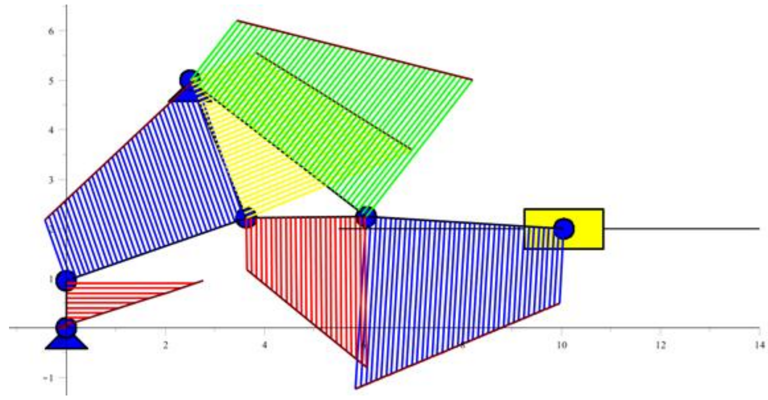

Figure 12: The investigating mechanism,
on which links the diagrams of longitudinal forces are constructed

6. CONCLUSIONS

The developed technique can be applied in the study of stress-strain state of the projected and existing mobile and fixed beam systems with statically definable structures (planar rod mechanisms, manipulators, frames, etc.).

REFERENCES

[1] Du Zhaocai, Yu Yueqing, Yang Jianxin. *Analysis of the dynamic stress of planar flexible-links parallel robots*. Vol. 2, Issue 2, pp 152-158. Higher Education Press and Springer-Verlag, 2007.

[2] Santosha K.D., Peter E. *Dynamic analysis of flexible manipulators: A literature review*. Mechanism and Machine Theory, 2006, 41: 749-777.

[3] Gaultier P.E. *Modeling of flexible manipulator dynamics: A literature survey*. 1st nat. Applied Mechanics Conference, Cincinnati, 1989, 2C-3: 1-10.

[4] Kang B, Mills J.K. *Dynamic modeling of structurally-flexible planar parallel manipulator*. Robotica, 2002, v20: 329-339.

[5] Utenov M.U. *Investigations of forces, arising from the self mass of links with constant and variable cross-sections in their plane-parallel motion*. Proceedings of the First International Scientific and Practical Conference, Vol. 2, pp 30-34. Almaty, 2000.

[6] Utenov M.U. *The matrix of approximation of element force under the action of distributed load with parabolic intensity*. Proceedings of the First International Scientific and Practical Conference, Vol. 2, pp 55-58. Almaty, 2000.

[7] Utenov M.U. *Construction of discrete models of planar rod mechanisms in the elastic calculation.* Bulletin of the Kazakh Academy of Transport and Communications. №6 [12].-2001,- pp 61-64.

[8] Chiras A.A. Structural mechanics. – M.: Stroyizdat, 1989. – 255 p.