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DEVELOPMENT OF ACCURATE PNEUMATIC TYRE FINITE ELEMENT MODELS BASED ON AN OPTIMIZATION PROCEDURE

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Abstract. *A novel method for extracting the geometric and constitutive material properties of pneumatic tyres from available numerical or experimental data for the development of realistic and reliable tyre numerical models is proposed. This method involves an optimization procedure, which incorporates a finite element model as a solver (ABAQUS) properly coupled with an optimiser function (MATLAB). Following that, an initial tyre model (P235/75R17) is developed, and then its properties are suitably adjusted via the optimization process, in order for the former to best fit a target model available in the literature, with respect to eigenfrequency analysis results. After the termination of the algorithm, the "optimum" tyre model (i.e. the model which best conforms to the target model) is obtained, the response of which is further investigated to ensure its realistic behaviour, which warrants its use for various numerical simulations. The results of this study show clearly the efficiency of the optimization procedure proposed, as well as the realistic response of the tyre model developed.*

1 INTRODUCTION

Handling low frequency interior noise and vibrations which transmits through subframe components on vehicles is a main issue regarding their design. The importance of this aspect is apparent from the related legislation which limits the level of noise a vehicle is allowed to produce. The main source of the vehicle noise is the vibrations induced by the tyres. These, after being transmitted from the tyre to the wheel axle, and through that to the passengers in the vehicle, can have various undesirable effects, some of which are the passengers' inconvenience or body distress, the low performance of the vehicle and its suspension system, etc.

A tyre is subjected to dynamic forces mainly from two main sources: (a) road surface irregularities, potholes, bumps and various other obstacles which impose dynamic loads to the tyre, and (b) dynamic loads originating from various nonuniformities of the tyre, such as slight imbalances or asymmetric tread pattern designs.

It is essential to consider the dynamic characteristics of the tyres of a vehicle, to minimise the aforementioned negative consequences. For this purpose, there is need for detailed knowledge of the dynamic response of a tyre, which is associated with the energy that is being transmitted to the vehicle from various external dynamic events. The dynamic response of a tyre is characterized by its vibration modes, or eigenmodes, namely the natural frequencies of the tyre and the corresponding mode shapes. These, apart from their significance for the design process and troubleshooting of various problems, can constitute a basis for the computational efficiency of the various numerical models of tyres used by both tyre and automotive industries for prediction of performance.

In this study the eigenmodes at the low frequency range are considered for the development of a realistic tyre model, based on numerical data published in the literature. This is achieved through an optimization process which efficiently adjusts various tyre parameters, so that the eigenmodes of the final tyre model reach the corresponding data as close as possible. After the optimum tyre properties are found, the configured model is then used to study the effect of inflation pressure and vertical load (i.e. stationary tyre loaded with vertical force) on its eigenmodes.

2 LITERATURE REVIEW

Tyre vibration modes are widely used over the years to represent dynamics in tyre models. The dynamic response of tyre models has been studied analytically, experimentally or semi-empirically, and numerically, however due to the limitations of the analytical and experimental studies, many studies in the literature employ numerical (often finite element) models, which can simulate complex geometries as well as material, geometric and boundary nonlinearities. In this study, the effect of the various parameters on the tyre response is incorporated into an optimization procedure, which ultimately determines the optimum values of these parameters, in order to minimise the error between the numerical model and the available data. Relevant studies about tyre dynamics, as well as optimization procedures are mentioned in the next.

2.1 Experimental studies

Experimental studies about the eigenmodes analysis of tyres can be found in [1-3]. In [1], the dynamic response of the vehicle in terms of accelerations was monitored at the wheel axis and the passenger compartment. The tyre vibration modes were identified from the peaks in the response. In [2] a lumped parameter model was developed to study the behaviour of a tyre running on a road surface with irregularities characterized by short wave-length spectrum components. However, the parameters of the lumped model are given by empirical relations, which have resulted from an experimental methodology. In [3] an experimental modal parameter estimation method is presented, in which the frequency response function (FRF) of a tyre is decomposed into the components of individual modes based on the Fourier transform algorithm.

2.2 Analytical studies

The analytical models developed for the estimation of the eigenproperties of a tyre range from simple mass/spring systems to various forms of idealized, spring supported, flexible rings. Representative studies are these in [4], where a rotating and vibrating tyre coupled at its spindle to a secondary structure is simulated. A model of a membrane on an elastic foundation is used for the description of the vibration of a rolling tyre, as well as models for the calculation of the forces at the spindle of a tyre rolling over a small cleat. In [5], the tyre is modelled as a shell structure in contact with the road surface. The contact patch is simulated as a prescribed deformation, and the coupled tyre-cavity governing equation of motion is solved analytically to obtain the tyre structural and acoustic responses.

2.3 Numerical studies

Numerical studies regarding the modal analysis of tyres can be found in [6-9]. In [6] the vibration modes of radial tyres on a fixed spindle can be seen and the effect of the tyre components and their contribution in the mode shapes is investigated. Following that, the corresponding tyre model under rolling conditions was considered in [7], and it was shown that non-rolling tyre models are subordinate to their rolling counterparts, as they do not take into account the proper kinematics. In [8], the finite element commercial software ANSYS was used to study the effects of the inflation pressure, the ply angle, the tread pattern and the thickness of the belt on the natural frequencies of the tyre. A basic assumption in this study was that the rubber was simulated as a linear elastic material. Another commercial finite element software (ABAQUS) has been used in [9], where by using various capabilities of ABAQUS, the footprint under purely vertical load was obtained for a vertically loaded tyre. Afterwards, the nodes (node coordinates) being in contact with the road were maintained in contact by applying an equivalent distributed vertical load, whereas the centre of the wheel was set free in all degrees of freedom. In this condition of the model, a frequency analysis was performed and it was found that the boundary conditions on the tyre model can have large impact on its eigenmode response.

2.4 Optimization

In general, the methods used to optimise a model (optimization methods) range from relatively simple mathematical programming based (exact) methods to novel heuristic search techniques. The methods belonging to the first category are very efficient for cases with a few design variables. Methods belonging to this category are those using the sequential quadratic programming procedure for nonlinear optimization (used in this study), as well as others. More details regarding these methods are presented in [10]. More robust optimization techniques, which are capable of searching effectively the whole design variable domain and not being trapped into local optima, can be used for increased number of design variables, or non-differentiable functions. Recently developed heuristic methods, such as genetic algorithms, simulated annealing, threshold accepting, tabu search, ant colonies, particle swarm, etc. provide more attractive alternatives.

3 NUMERICAL MODELING

3.1 Introduction

The tyre considered in this study was modelled in the commercial finite element code ABAQUS 6.13. Implicit integration was performed using ABAQUS/Standard, which was also used for the eigenfrequency and eigenmode extraction of the tyre. The optimization procedure, as well as the necessary coupling with ABAQUS, was implemented in MATLAB programming language.

3.2 Tyre modeling

The cross section of the tyre, P235/75R17, is shown in Figure 1. The tyre is comprised of the belt region, the tread region and the side walls which are being modelled with a hyperelastic material, representative of rubber. The hyperelastic material is simulated by the one term polynomial strain energy potential (Mooney-Rivlin model) with one term Prony series to account for viscoelasticity [11]. The belt region contains reinforcement of two layers (illustrated as Belt layer 1 & 2 in the Figure), and the reinforcement of carcass. The last extends over the belt region and it covers the side walls. Both belt layers and the carcass are discretized with surface elements with twist (SMFGAX1). The rim is discretized with 2-node, linear links for axisymmetric planar geometries (RAX2), and the belt, bead, sidewall and tread regions are discretized with 4-node bi-linear, reduced integration elements with hourglass control (CGAX4R). The nodes of the surface elements of the carcass share the same nodes with those of the belt region elements. If separate nodes are used for these two sections (which have the same coordinates) numerical instabilities may occur during the analysis.

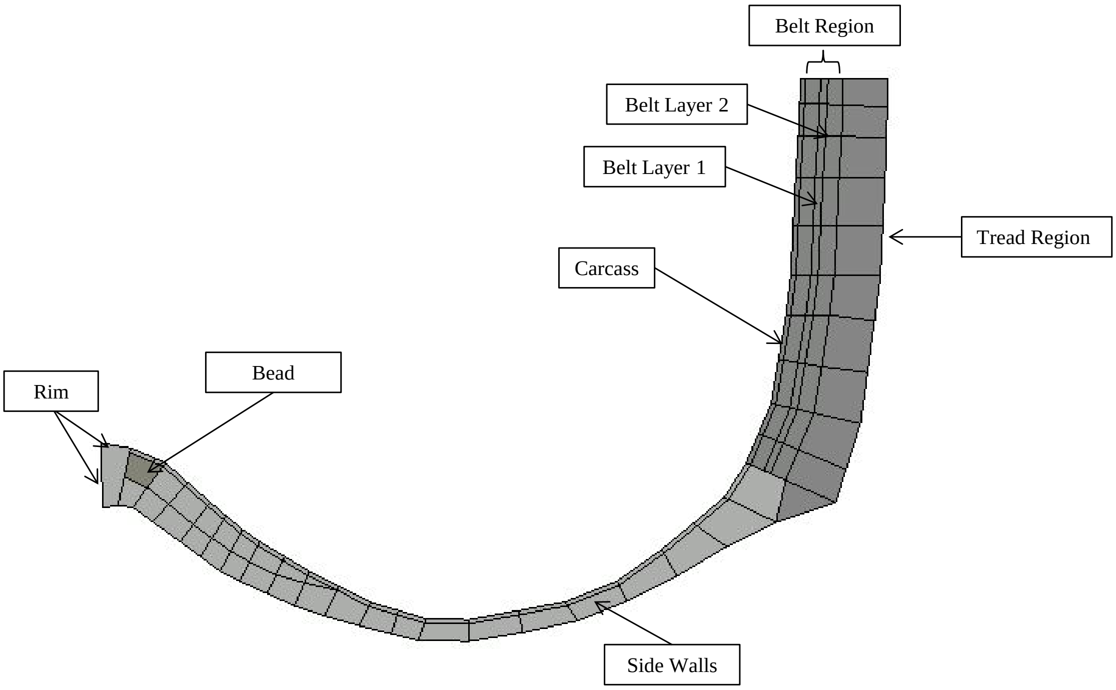


Figure 1: Tyre half-cross section geometry.

By utilizing the capabilities of ABAQUS with regard to symmetric model generation (SMG), symmetric results transfer (SRT) and restart option, the full 3d numerical model of the tyre is developed, as shown in Figure 2. Inflation pressure is imposed on the inner surface of the tyre as a distributed load. Regarding the boundary conditions, two cases can be distinguished: (a) for the unloaded tyre, the boundary conditions are imposed on the six degrees of freedom of the wheel centre (fixed-spindle), and (b) for the loaded tyre, the road is considered to be fixed and the tyre centre is constrained along all degrees of freedom except for the degree of freedom along which the vertical load is imposed. The rim is rigidly constrained to the tyre centre. The friction between the tyre and the road (in the case of the loaded tyre) is assumed to be of Coulomb type, with coefficient equal to 0.5.

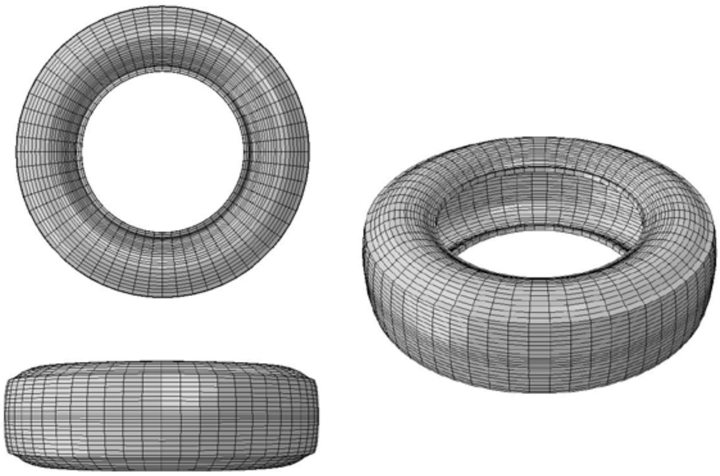


Figure 2: Illustration of the tyre model.

3.3 Formulation of the optimization problem

In this section, the optimization procedure is outlined, and its various aspects are presented (design variables, parameters, constraints and objective function).

- Design variables

The geometric properties of the belts and carcass reinforcement, as well as the hyperelastic Mooney-Rivlin C_{10} constant are selected as design variables. The reinforcement layers are defined in ABAQUS as smeared layers with a thickness equal to the ratio of the area of each reinforcing bar to the reinforcing bar spacing. This calculated thickness is assumed to remain constant all over the extent of the layer. This consideration has a considerable effect on the selection of the design variables, since the stiffness of each reinforcement layer contributes to the eigenproperties of the tyre. Due to the fact that the rebar stiffness is given by a fraction of two separate input parameters, for constant layer stiffness they become dependent on each other. Therefore, it is objective that only one of the two parameters for each layer is selected as an independent design variable, and the other remains fixed. The variable to remain fixed is the easier to be measured, in terms of order of magnitude. Another point to be mentioned is that, because the two belt layers have symmetric orientation with respect to the plane of the tyre, and the tyre is a centre symmetric structure, its eigenmodes are expected to be also symmetric; this means that the cross section areas of the two belt reinforcements have to be equal, and therefore the belt reinforcement cross sectional area was considered as a single design variable. The design variables of the optimization problem, along with their upper and lower bounds are shown in Table 1.

Design variable	Lower bound	Upper bound
A_{belt}	10^{-7}	10^{-5}
$A_{carcass}$	10^{-8}	10^{-5}
C_{10}	10^{5}	10^{7}

Table 1: Design variables of the optimization problem and their lower and upper bounds.

- Parameters

The parameters of the optimization problem are the design input data that remain fixed during the optimization process. These include, as aforementioned in the previous section, the spacing of the rebar layers, which is set to be equal to 0.00116m for the belts and 0.001m for the carcass. Furthermore, the constants of the Mooney-Rivlin strain energy potential are C_{01}=0 and D_1=5.085*$10^{-8}$$Pa^{-1}$. The cord angles are 70 and 110 degrees for the two belt layers, and 0 degrees for the carcass. The material properties of the belts and the carcass are also held fixed during the optimization process. More details about these properties can be found in [11]. The inflation pressure with which the tyre is inflated is 240kPa.

- Constraints

No constraints are imposed to the model being optimised, apart from the upper and lower limits of the design variables. The latter require some experience to be specified, because large upper bounds or small lower bounds can lead to numerical instabilities in the solver, such as excessive element distortion, etc, which result in the premature termination of the optimization procedure.

- Objective function

The objective function for the optimization problem has to be of an appropriate form, so that it becomes minimum if the numerically calculated eigenfrequencies coincide with the ones available from the literature. The first 16 eigenfrequencies of the tyre are considered in the objective function, which is given by the equation:

$$obj = \sqrt{\sum_{i=1}^{16}\left(f_{i,num} - f_{i,lit} \right)^2} \quad (1)$$

where $f_{i,num}$ is the i^{th} eigenfrequency calculated by the numerical model in every iteration of the algorithm and $f_{i,lit}$ is the corresponding i^{th} eigenfrequency available in the literature. The correspondence between the various eigenfrequencies is made by taking into account the deformed configurations of the various eigenmodes.

3.4 Algorithm used for the optimization problem

The optimization algorithm used in this study is a sequential quadratic programming (SQP) method. In this method, a quadratic programming (QP) subproblem is solved at each iteration. For this purpose the MATLAB built in function fmincon is used. This function used an active set strategy and updates an estimate of the Hessian of the Lagrangian at each iteration using the BFGS formula. An active-set method initializes by making a guess of the optimal active set, and if this guess is incorrect, it repeatedly uses gradient and Lagrange multiplier information to proceed towards the optimum solution.

The fmincon optimiser (MATLAB) is properly coupled with the analysis solver (ABAQUS) in order to take the frequency analysis results. This is done inside the objective function in which ABAQUS is called to perform the necessary analyses. Except for this, the necessary input (*.inp) files for the ABAQUS runs are created by suitable MATLAB functions. To read the results of the analyses from the corresponding ABAQUS results (*.fil) files, special MATLAB functions are used. While the analysis solver is running the optimiser is halted and its execution is continued after the lock (*.lck) file has been deleted.

4 OPTIMIZATION RESULTS

The results of the optimization process as presented in the previous sections are shown in Table 2.

	Initial model	**Optimised model**	**Wheeler et al. [6]**	**Deviation (%)**
		Design Variables		
A_{belt} (m^2)	$2.11868*10^{-7}$	$3.64826*10^{-7}$	N/A	-
$A_{carcass}$ (m^2)	$4.20835*10^{-7}$	$8.01133*10^{-8}$	N/A	-
C_{10} (Pa)	10^6	10^6 +0.01489	N/A	-
		Eigenfrequencies		
f_1 [0,0] (Hz)	36.85	30.86	31.7	2.66
f_2 [0,0] (Hz)	37.17	35.85	35	2.43
f_3 [1,1] (Hz)	43.85	36.92	37.8	2.33
f_4 [1,1] (Hz)	43.85	36.92	37.8	2.33
f_5 [1,0] (Hz)	65.07	58.75	58.5	0.43
f_6 [1,0] (Hz)	65.07	58.75	58.5	0.43
f_7 [2,1] (Hz)	76.33	68.41	66.1	3.49
f_8 [2,1] (Hz)	76.33	68.41	66.1	3.49
f_9 [2,0] (Hz)	86.65	78.67	79.5	1.04
f_{10} [2,0] (Hz)	86.65	78.67	79.5	1.04
f_{11} [3,0] (Hz)	104.36	96.42	97.6	1.21
f_{12} [3,0] (Hz)	104.36	96.42	97.6	1.21
f_{13} [3,1] (Hz)	117.07	107.9	102.7	5.06
f_{14} [3,1] (Hz)	117.07	107.9	102.7	5.06
f_{15} [4,0] (Hz)	122.65	114.9	115.9	0.83
f_{16} [4,0] (Hz)	122.65	114.9	115.9	0.83
		Algorithm Details		
Min. value of obj. function	-	8.59	-	-
Number of obj. function evaluations	-	25	-	-

Table 2: Results of the optimization procedure of the tyre frequency analysis considered in this study.

It is noted that each natural frequency corresponds to a pair of integers enclosed in brackets ([c,m]). The first integer denotes the number of sinusoidal waves in the circumferential direction of the wheel, whereas the second integer shows the number of waves in the meridional direction at a specific location, where the deformation of the eigenmode shape is maximum. In addition, only the first 16 eigenmodes were considered for the development of the realistic tyre model, in order to reduce the computational cost.

The first column of Table 2 shows the data of the initial model, used as the starting point of the optimization process. It is evident that the eigenfrequencies of the initial model have large difference from the eigenfrequencies of the model published in [6]. In the second column, the parameters of the optimum model are shown, as well as the values of the design variables leading to it. Regarding the eigenfrequencies, it is observed that they are much closer than those of the initial model, leading thus to a numerical model that conforms more to the available numerical data, and therefore it is more realistic. The maximum deviation of the eigenfrequencies is noted to be roughly 5%. The optimum model has higher cross section of the reinforcement of the belts, and lower cross section area of the reinforcement of the carcass than the initial model. The hyperelastic constant C_{10} is only slightly increased after the optimization. Regarding the algorithm output, the minimum value of the objective function is equal to approximately 8.59Hz, and the algorithm converged after 25 objective function evaluations. The reason for the termination of the algorithm is that the magnitude of the search direction was less than the corresponding tolerance. The most important factor affecting the tyre modal behaviour during the optimization procedure is proved to be the cross section area of the carcass ($A_{carcass}$). Due to the fact that the initial model has generally higher eigenfrequencies than those of the target model [6], its stiffness had to be decreased, in order for the model to approach the latter. The decrease in stiffness is achieved with a relatively large decrease in the cross sectional area of the carcass, although the cross section area of the belt reinforcement increases.

In Figure 3 the various eigenmodes of the optimised tyre model are shown. The figure is divided into 9 subfigures, each of which shows a tyre eigenmode shape viewed from 4 different perspectives. The fundamental eigenmode is the axial or lateral mode, and after this the torsional, pitch, diametric, and higher modes follow. There is total correspondence between the integer pairs which appear in the bottom of each subfigure, and the ones shown in the first column of Table 2.

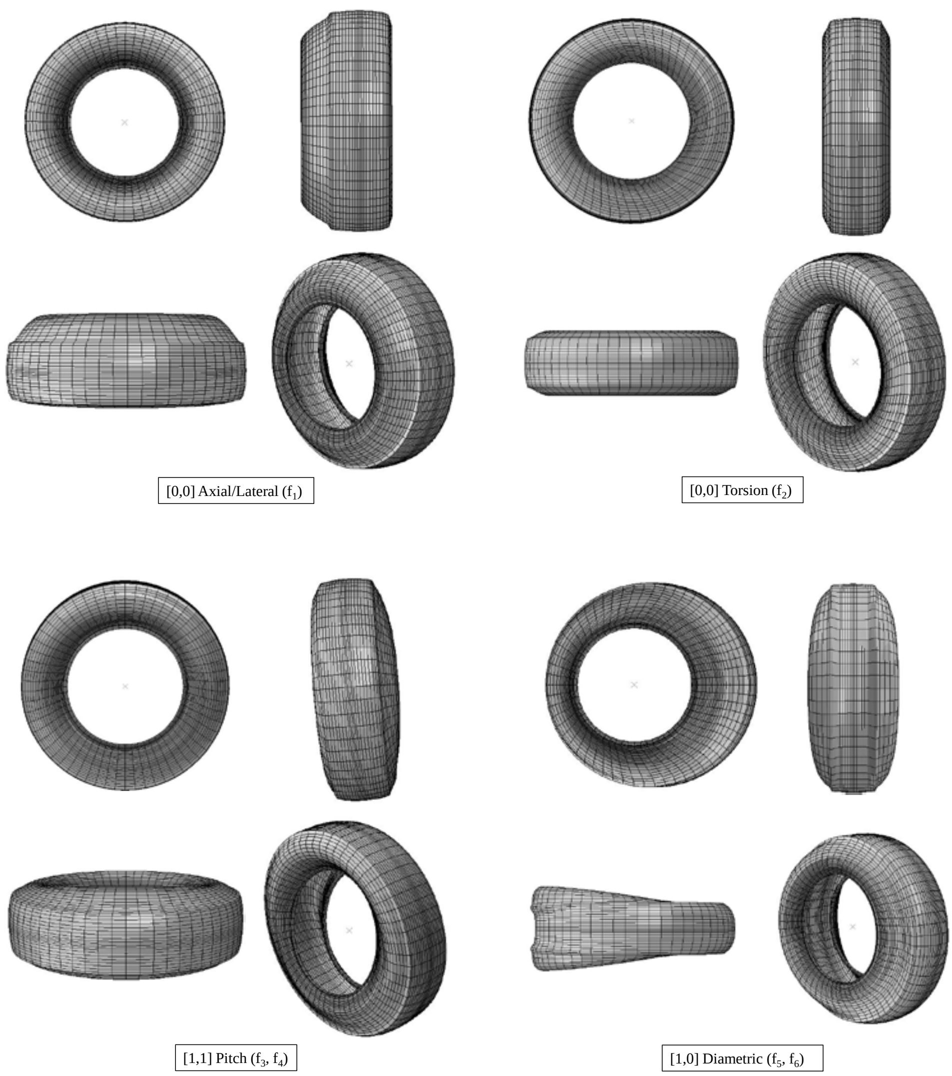
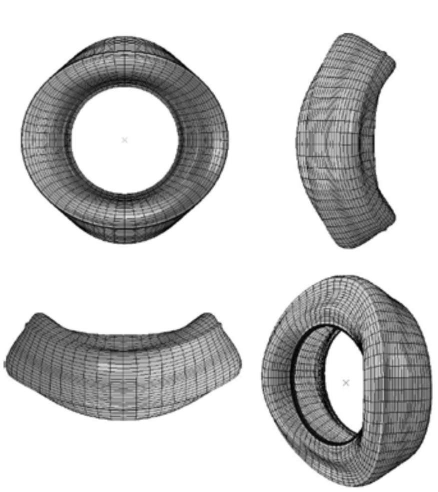
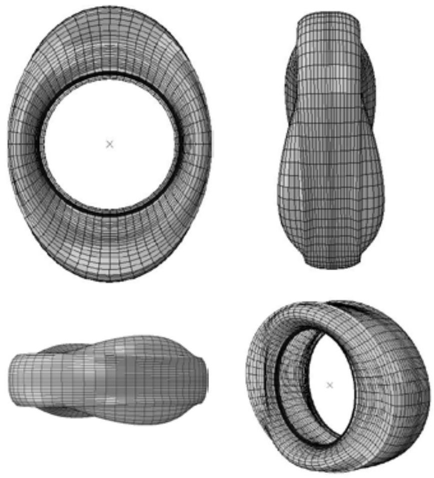


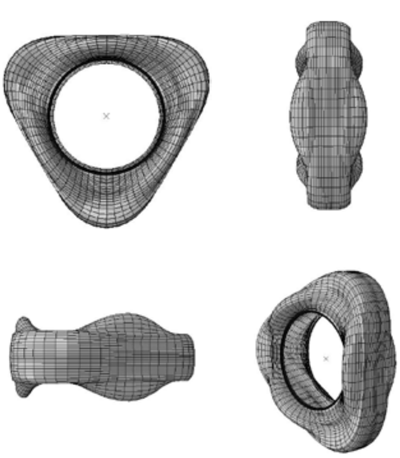
Figure 3: Eigenmode shapes of the optimised tyre model (continued in the next page).



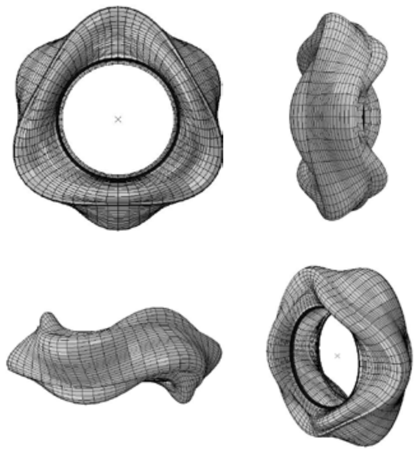
[2,1] (f_7, f_8)



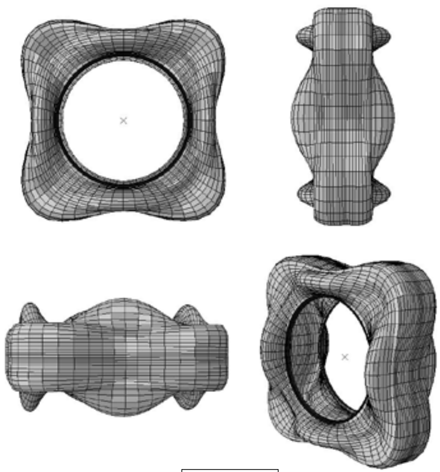
[2,0] (f_9, f_{10})



[3,0] (f_{11}, f_{12})



[3,1] (f_{13}, f_{14})



[4,0] (f_{15}, f_{16})

Figure 3: Eigenmode shapes of the optimised tyre model (continued from previous page).

5 EFFECT OF INFLATION PRESSURE ON MODAL RESPONSE OF OPTIMUM TYRE

The optimised model is the most realistic version of the selected tyre type (P235/75R17) with respect to its modal response. It is close to the modal data available in [6], and for this reason, it allows its use for dynamic response analyses. In an attempt to further valildate the optimised model, the variation of its eigenmodes and eigenfrequencies is studied for varying inflation pressure.

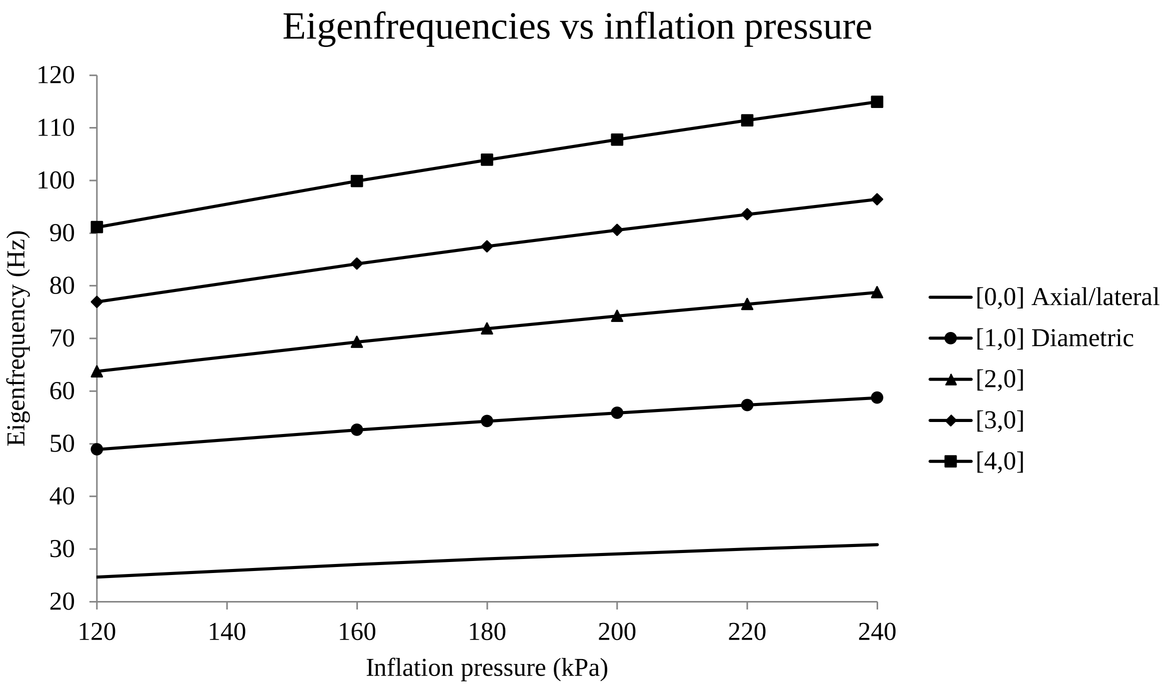


Figure 4: Eigenfrequencies of various mode shapes of the optimised tyre model for various inflation pressures.

In Figure 4 the effect of the inflation pressures on the eigenfrequencies of the tyre can be observed. As it can be expected, as the inflation pressure rises, the eigenfrequency of a specific eigenmode increases, as the increased inflation pressure makes the tyre stiffer. This is a trend widely observed in the literature and once again corroborates the realistic behaviour of the optimum tyre. Moreover, it is apparent that the increase of the eigenfrequency of each mode for increasing inflation pressure is nonlinear. Specifically, for lower values of the inflation pressure, the rate of increase in the eigenvalues becomes higher than that for higher values of the inflation pressure. Finally, for the higher order eigenmodes, the increase in the eigenfrequency for the same difference in the inflation pressure is larger than that for the lower eigenmodes, which is in agreement with relevant results found in [8].

6 COMPARISON BETWEEN OPTIMUM AND INITIAL TYRE MODELS

In this section the static response of the initial tyre model and the optimised tyre model is considered with a view to make a comparison between the two models in terms of their response. For this purpose, the contact area of a vertically loaded tyre on a rigid surface is calculated for the two tyre models. The results are shown in Figure 5. It seems that the contact area of the optimised tyre model is generally larger than that of the initial tyre model, for the same values of vertical load, with an exception in the interval of vertical load around 2 kN, where the two areas are roughly equal. This is a direct consequence of the fact that, the eigenfrequencies of the optimum tyre model are lower than the corresponding ones of the initial tyre model, which indicates lower stiffness, and therefore larger contact area. Although the difference seems to be relatively small for vertical loads lower than 2 kN, the discrepancy is expected to be quite significant for vertical loads higher than 4 kN. The values of the contact area calculated for the optimum model are close to numerical [12] and experimental [13] results found in the literature which proves that this tyre model exhibits realistic behaviour.

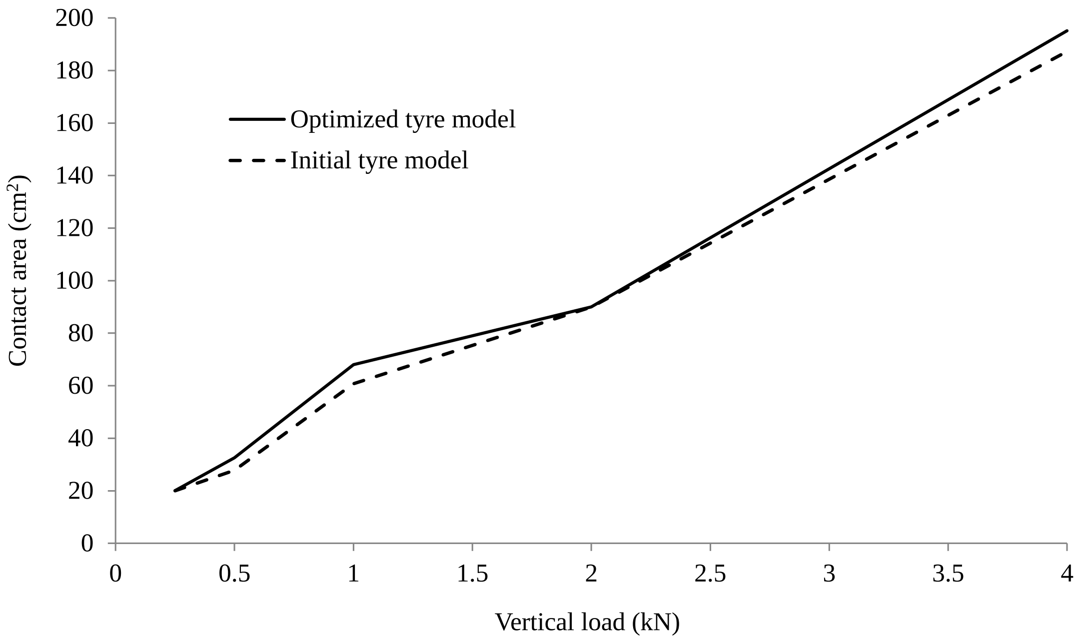


Figure 5: Contact area versus vertical load for the two tyre models considered in this study.

7 CONCLUSIONS

- An optimization procedure has been implemented which successfully adjusted the geometric properties of a tyre (P235/75R17), so that its modal properties match closely those available in the literature [6].
- The optimization procedure proposed in this study can be extended for the calibration of various tyre material constitutive models, which in most cases are unknown, based on experimental data, such as contact area and modal analysis results. For instance, the parameters of the Mooney-Rivlin hyperelastic material model can be extracted from tyre response data through the optimization procedure. In general, this procedure can be followed for the development of any realistic tyre model.
- Successful coupling between the finite element software ABAQUS (used as the solver) and MATLAB (used as the optimiser) was performed, in a way that optimises the running speed of the whole optimization process.

- The deviation of the eigenfrequencies of the optimised tyre model from the corresponding eigenfrequencies of the target tyre model [6] (i.e. the model to which the initial model is fitted) was generally small, not larger than roughly 5%.
- The response of the optimised tyre model was found to be in close agreement with available numerical and experimental data.

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