A MIXED OPERATOR OVERLOADING AND SOURCE TRANSFORMATION APPROACH FOR ADJOINT CFD COMPUTATION

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Abstract. Adjoint based calculation of sensitivities pertaining to a Computational Fluid Dynamics (CFD) Solver has proven to be a vital tool in tackling large scale problems often found in industrial applications. There are two basic approaches for applying Algorithmic Differentiation (AD) to a CFD solver, namely, Operator Overloading and Source Transformation. Unfortunately, in both cases, it is still necessary to invest a significant amount of manual coding in order to get to an application that performs acceptably in terms of memory consumption and runtime. In this paper reverse mode of AD has been applied to an unstructured pressure-based steady Navier-Stokes solver. We explore the feasibility of combining both kinds of AD approaches to show where and how the advantages of each method can be exploited in order to reach a suitable compromise between performance, simplicity and efficiency. Additionally, we propose a methodology to automate the implementation of an adjoint software in order to minimize work the developer must carry out to produce the desired derivative. We investigate the effectiveness of this approach for relevant flow test cases.
1 INTRODUCTION

Algorithmic Differentiation (AD) \[1, 2\] employs the rules of differential calculus in an algorithmic manner to determine accurate derivatives of a function defined by computer programs. For a given implementation of the flow model \( F : \mathbb{R}^n \rightarrow \mathbb{R}^m \) over a computational grid, the computer program is developed to simulate the functional dependence of one or more objectives \( y \in \mathbb{R}^m \) on a potentially large number of input variables \( x \in \mathbb{R}^n \) by simulation of:

\[
F : \mathbb{R}^n \rightarrow \mathbb{R}^m, \quad y = F(x)
\] (1)

AD enables us to compute the corresponding derivatives \( \frac{\partial y}{\partial x} \) in forward (forward mode) or backward (reverse mode). For a given implementation of the primal function in Equation 1, the function \( F_1 : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n \times \mathbb{R}^m \), defined as

\[(y, x_1) = F_1(x, y_1) \equiv \left( F(x), \left( \frac{dy}{dx} \right)^T \cdot y_1 \right)\]

(2)
is referred to as the adjoint model of \( F \). The adjoint model implementation yields the objective \( y \) and the product \( x_1 \equiv \nabla F(x)^T \cdot y_1 \) of its transposed jacobian at the current point \( x \in \mathbb{R}^n \).

There are two main methods for implementing AD: by source code transformation (S-T) or by use of derived data types and operator overloading (O-O). In O-O AD the code segments and arguments of the primal code are stored inside a memory structure called tape during the forward run of the primal. In reverse mode the stored values on the tape are interpreted to get the resulting adjoints, while the S-T approach parses the code at compile time and generates the actual derivative code.

Prior to this paper \[3, 4\], we have discussed the implementation of the AD tool \texttt{dco/fortran} \[^1\](Derivative Code by Overloading in Fortran) to an unstructured pressure-based steady Navier-Stokes solver. The solver is an incompressible flow solver with cell-centered storage and face-based residual assembly. It works on unstructured meshes with collocated variables and uses the SIMPLE \[5\] pressure correction algorithm in a pseudo time stepping scheme towards a steady solution. It is written in Fortran 90-95 (7,000 lines) as a test-bed for developing adjoint Navier-Stokes fields \[6\]. The case study that is used for flow model simulation and sensitivity studies is the S-bend channel flow case which is a simplified vehicle climatisation duct that is presented in \[7\]. We have addressed proper solution algorithms adapted to the code for the improvement of efficiency of the adjoint code by optimizing the checkpointing scheme for the iterative solver and development of symbolically differentiated of linear solver. In addition, we investigated the benefit of the reverse accumulation technique \[8\] for the fixed point iterative construction in the primal code.

In this paper we combine the flexibility and robustness of operator overloading with the efficiency of source transformation by coupling \texttt{dco/fortran} and \texttt{TAPENADE} \[9\]. The latter is used for the derivation of computationally expensive kernels. Our emphasis is to automate the implementation of an adjoint software to decrease the development time of the differentiated code. The overall design is presented in Sections \[3\] and \[4\] illustrating the transformation procedure. Numerical results demonstrate the computational efficiency in terms of memory

\[^1\]developed at the institute \textit{Software and Tools for Computational Engineering} at RWTH Aachen University implementing AD by overloading in Fortran \[1\].
consumption and speed while preserving the flexibility of an O-O approach applied to a CFD code.

2 REVIEW OF AD TOOLS

Algorithmic Differentiation is generally implemented using one of two strategies: source code transformation or operator overloading. Here we present a brief review of both tools.

2.1 Operator overloading

Using AD by operator overloading to evaluate directional derivatives of an existing program is comparatively easy. The main idea is to replace all relevant floating point data types within a given program with corresponding enhanced types of the particular operator overloading tool. The enhanced data types then keep track of any tangent- or adjoint-information of the specific variables declared as floating point data types throughout the forward execution of the program. This is accomplished by defining special operators for all intrinsic functions like +, -, sin(), exp(), etc., hence the name operator overloading. In case of a tangent-linear model derivative information is acquired alongside the forward execution of the program. Using the adjoint model this is not the case. When an intrinsic function is called during the forward execution of the program, the required information to compute the adjoint information of all involved active (i.e., tracked) variables does not yet exist. To show that, consider the assignment \( y = x_1 \cdot x_2 \). The adjoints for both inputs are \( x_1^{(1)} + = y^{(1)} \cdot x_2 \) and \( x_2^{(1)} + = y^{(1)} \cdot x_1 \) respectively. Unfortunately, the value for \( y^{(1)} \) is not yet available when the assignment is executed. Therefore, the derivative information cannot be computed at that point in time. A two step approach to solve this problem first stores all required information for every operation during the forward execution of the program. The second step is running through the operations in reverse order and calculating adjoint values of its inputs \( (x_1, x_2) \) from adjoint values of its outputs \( y^{(1)} \) which are now available. Storing of such information is usually realized by creating log entries on a stack also referred to as tape. In case of dco/fortran, an entry in such a tape stores a code for the type of operation, virtual addresses of operands \( x_1 \) and \( x_2 \) as well as floating point values for its value \( y \) and adjoint \( y^{(1)} \). This method automatically fulfils requirements of data-flow and control-flow-reversal for an adjoint model without any extra effort by the tool or the developer. Understandably, such an approach may require a very large amount of available memory in order to store the entire tape for programs of even small to moderate complexity. Its big advantage is its ease of implementation. There are different O-O AD tools for Fortran programs such as ADF [10], ADOL-F [11], AUTO_DERIV [12], etc.

2.2 Source transformation

AD by source transformation analyses the source code of a particular function and produces a new source code which, when executed, calculates the original function value and its directional derivative. The source code for a function is replaced by an automatically generated source code that includes statements for calculating the derivatives interleaved with the original instructions. The advantage is that the resulting code can be compiled into an efficient program due to compile time optimizations having a greater effect. This is a valid approach for all programming languages. The drawback is that this is an enormous transformation that usually cannot be done by hand. Tools are needed to perform this transformation correctly and rapidly. For a tangent-linear model, the transformation is straightforward because the derivative statements are in the same order as the original statements. The adjoint model requires control flow reversal which
turns it into a challenging problem for S-T tool developers. Tapenade [9] engine is just one such Automatic Differentiation tool that uses source transformation. Other examples of S-T tools that provide automatic differentiation for Fortran programs are ADG [13], ADFOR [14], TAF [15], OPENAD [16].

The relative advantages and disadvantages of using the two different tools are summarized in Table 1. The + sign indicates that the corresponding AD tool possesses the desired characteristic.

<table>
<thead>
<tr>
<th>AD tool</th>
<th>O—O</th>
<th>S—T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ease of implementation</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Changing the original source code</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Memory consumption</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>Compile time optimizations</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>Compatibility of numerical gradients with the discrete PDE</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Flexibility to handle arbitrary functions</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>Robustness</td>
<td>+</td>
<td>−</td>
</tr>
</tbody>
</table>

Table 1: Comparison operator overloading and source transformation AD tools

3 MIXED APPROACH: SOURCE TRANSFORMATION VIA OPERATOR OVERLOADING TOOL

We apply S-T via O-O tool using a feature of dco/fortran called External Function. This interface provide adjoints of specific functions of a program to dco/fortran or TAPENADE. It is typically used to provide analytically derived adjoints for computationally intense kernels. Instead of recording the operations of such a kernel during the forward run a function pointer is registered on tape. It points to a function capable of providing the adjoint information of the kernel. Basically, if the adjoint model of a function \( f \) is known, this information can be used during the interpretation in order to remove the necessity to record \( f \) during the forward execution of the program. Depending on the complexity of \( f \), this may have a drastic impact on memory consumption. The following small example shows how this feature is used in order to mix different approaches to adjoint computation.

Consider a function \( z = g(x, p) \) with \( x \in \mathbb{R}^n \) and \( z \in \mathbb{R}^m \) which at any point during its execution calls \( y = f(x) \) [1]. Let \( f_{(1)} \), described in (2), be its known first order adjoint model and let \( P \) be a program that computes the adjoint \( z_{(1)} \) of \( g \) using dco/fortran. Including \( f_{(1)} \) into \( P \) takes two function implementations. First, a function \( f_{\text{wrap}}(x) \) that hides the process described in figure [1] denoted by X-Forward from the rest of \( P \). Second, a function \( f_{\text{ext}} \) which includes the steps X-Reverse and calls the desired implementation of \( f_{(1)} \) in between steps 3 and 4.

1-Forward The state of every non-constant variable which is needed to evaluate \( f \) has to be saved. In addition, independent inputs for \( f \) are registered using the checkpointing features of dco/fortran in order to have the correct variable addresses available later in step 4.

2-Forward The output of \( f \) is saved and the implemented function \( f_{(1)} \) is registered as the external function that is later called by the interpretation process to evaluate adjoints \( x_{(1)} \).
1-Forward:
- save x
- register inputs x

2-Forward:
- save y
- register external function f_ext(.)

3-Reverse:
- load x, y
- extract adjoint y_{(1)}

4-Reverse:
- increment adjoints x_{(1)}
- compute y
- compute x_{(1)}

Figure 1: External Function Concept

**3-Reverse** The stored data is loaded and the adjoint y_{(1)} is read from tape.

**4-Reverse** The computed adjoint information has to be added back into the tape at their corresponding positions. This is done by incrementing adjoint values of all registered variables in reverse order.

4 AUTOMATION METHODOLOGY

The idea of the automation process described here assumes that there already exists an overloaded program derived by using dco/fortran. This section lists steps that need to be taken in order to produce a combined (dco/fortran and TAPENADE) solution. The overview of the method is presented in figure 2. In addition, a basic approach for automatic source code generation of wrapper- and external functions is presented in 4.2.1 and 4.2.2 respectively. This section uses some abbreviations for the different versions of a function:

<table>
<thead>
<tr>
<th>f</th>
<th>The original overloaded function of the black-box dco/fortran version</th>
</tr>
</thead>
<tbody>
<tr>
<td>f_passive</td>
<td>The original function of the original program</td>
</tr>
<tr>
<td>f_b</td>
<td>The adjoint function derived by TAPENADE</td>
</tr>
<tr>
<td>f_wrap</td>
<td>The wrapper function corresponding to f</td>
</tr>
<tr>
<td>f_ext</td>
<td>The function called by the interpreter to compute f_{(1)}</td>
</tr>
</tbody>
</table>

Table 2: Function abbreviations

4.1 Kernel identification

The first and vital step is to choose the correct functions for extraction. Using a profiling tool like gprof[17] or intel’s vtune[18] is more than sufficient to provide a list of computationally
expensive functions. Another important information is how often those have been called during one execution of the original program. The reason for collecting that particular information is that using the external function feature involves storing/copying of data. This overhead may negate any benefit when the computational effort per function call is too small. Thus, the optimal candidates for extraction are functions with a significant self time and minimal number of calls. In addition, it is beneficial if such a function is low in the hierarchical calling order so as to avoid passing the entire program on to TAPENADE. Once such functions are identified they can be passed on to TAPENADE in order to generate the corresponding overloaded versions.

4.2 Code generation

This sections describes how the development of wrapper- and external functions can be automated. The focus is on source code generation. Not considered is the automated integration of newly created functions into a build system due to the variety in which such a system can be realized.

4.2.1 Wrapper generation

A Wrapper function \( f_{\text{wrap}} \) of an arbitrary function \( f \) has a fixed structure outlined in Algorithm 1. Due to this fixed structure, it is possible to define the complete source code of \( f_{\text{wrap}} \) using only a few additional informations about the arguments of \( f \). Once all required information is collected a Python preprocessor could generate wrappers, register routines and replace the code. The source code is determined knowing the following information about each variable appearing in the argument list of the function \( f \):

1. Input/Output/Parameter
2. Array Dimension(s)

3. Active/Basic/Derived Type

An active type is a specific derived type, namely `dco_als_type`. Knowing the input/output status of an active variable defines whether it needs to be registered as such. It also determines when its value has to be saved. Inputs and parameter values are saved before calling \( f \) whereas outputs naturally have to be saved after their evaluation. Array dimensions determine the loop structures encasing calls of functions like `register_input`. All basic types are classified as parameters by default and only their current values need to be stored. A special case occurs when processing derived types. This is due to the fact that a derived type may hold all kinds (input/output/parameter) of variables inside. Consequently, this information needs to be collected for all members of a derived type independently. In addition, a derived type may also have a member which is a derived type itself. A program/script capable of processing arbitrary types therefore has to have a part handling nested derived types. One way to realize such a program is to split it in two parts. The first part collects all required information and fills a data structure similar to Table 4 for each argument and each basic/active member variable of any derived types appearing in the argument list of \( f \). Then, for each step of Algorithm 1 it is checked for each variable if code has to be added to the current section corresponding to the current variable. This can be done by adding references of all `var` structures into three lists, `inputs`, `outputs` and `parameters`, depending on their `var.intent` value.

---

**Algorithm 1:** wrapper function generation

Input:
- module dependency information of the overloaded adjoint function \( f_1 \)
- three lists of `var` structs: `inputs`, `outputs`, `parameters`

Output:
- source code of function \( f_{\text{wrap}} \)

**Algorithm:**
genenerate function head
foreach `var in inputs` do
  | generate block `register input`
end
foreach `var in inputs & parameters` do
  | generate block `checkpoint`
end

generate call of original function \( y = f_{\text{passive}}(x,p) \)
foreach `var in outputs` do
  | generate block `checkpoint`
end
foreach `var in outputs` do
  | generate block `register output`
end

generate closure statements
4.2.2 External function generation

Like the concept of the wrapper function, the external function also follows a specific sequence of tasks that need to be executed in a fixed order. The sequence is presented in algorithm 2. The information required to generate external function code is equal to that of a wrapper function when making one simplification. Every array is assumed to be allocatable, meaning that all data structures are allocated when loading the checkpointed data in \( f_{\text{ext}} \). Not having this assumption just means adding a allocatable flag to the \Var \structure and distinguishing the two cases during code generation.

### Algorithm 2: external function generation

**Input:**
- module dependency information of \( f_{\text{passive}} \)
- three lists of var structs: inputs, outputs, parameters

**Output:**
- source code of function \( f_{\text{ext}} \)

**Algorithm:**

generate function head

\begin{verbatim}
foreach var in inputs & parameters do
    generate block read checkpoint
end

foreach var do
    generate block allocate adjoint
end

foreach var in outputs do
    generate block get_input_adjoint
end

generate call to \( f \)

foreach var in inputs do
    generate block increment_adjoint
end

generate closure statements
\end{verbatim}

4.3 Data collection

The collection of necessary information can be done in many ways. The method proposed here consists of parsing the output of gfortran -fdump-parse-tree and the result of the data flow analysis of TAPENADE. The latter is necessary for the identification of in- and outputs of the function where as the former is used to fill data type and shape information. The TAPENADE data flow analysis can be found on top of the source code of the derived function. For example, let a function \( f_{\text{passive}}(x,y) \) implement \( y = \sum_{i=1}^{n} x_i^2 \). The corresponding analysis of TAPENADE produces the following comments on top of the derived function \( f_{\text{b}}(x,xb,y,yb) \):

\begin{verbatim}
! Differentiation of \( f_{\text{passive}} \) in reverse (adjoint) mode (with options noISIZE):
\end{verbatim}
Table 3: Intent states of active variables for f_b

<table>
<thead>
<tr>
<th>State</th>
<th>meaning of var(1) for f_b</th>
<th>meaning of var for f</th>
<th>code blocks for f_wrap</th>
</tr>
</thead>
<tbody>
<tr>
<td>:zero</td>
<td>output, var(1) := 0</td>
<td>parameter</td>
<td>save_in</td>
</tr>
<tr>
<td>:incr</td>
<td>input, var(1) incremented</td>
<td>input</td>
<td>rgstr_in, save_in</td>
</tr>
<tr>
<td>:in-zero</td>
<td>input, var(1) := 0 at end</td>
<td>output</td>
<td>rgstr_out, save_out</td>
</tr>
<tr>
<td>:in-out</td>
<td>input &amp; output</td>
<td>input &amp; output</td>
<td>rgstr_in, rgstr_out, save_in</td>
</tr>
<tr>
<td>:out</td>
<td>output, var(1) := 0 at start</td>
<td>input &amp; output</td>
<td>rgstr_in, save_in</td>
</tr>
</tbody>
</table>

Table: Members of Var struct

<table>
<thead>
<tr>
<th>Var</th>
<th>string</th>
</tr>
</thead>
<tbody>
<tr>
<td>name</td>
<td>string</td>
</tr>
<tr>
<td>type</td>
<td>integer</td>
</tr>
<tr>
<td>rank</td>
<td></td>
</tr>
<tr>
<td>shape</td>
<td>[dim1, dim2, ..., dim_rank]</td>
</tr>
<tr>
<td>intent</td>
<td>{'in', 'out', 'in-out', 'parameter'}</td>
</tr>
</tbody>
</table>

5 NUMERICAL RESULTS

Numerical results are described for the S-bend test case in three dimensions for different problem sizes ranging from 47k up to 500k cells. Sensitivity analysis results of the adjoint code using operator overloading have been presented in earlier work [4] to calculate the sensitivity of the pressure loss objective function with respect to surface boundary nodes of the S-bend. These results have been obtained for the whole iteration process for the steady solver up to the convergence point of the flow solver. Applying only operator overloading to the flow solver leads to significant memory usage presented in Figure 3 (on tape). The performance has been improved by symbolically differentiating the linear solver. However, this is only applicable to problems where the function to be differentiated is simple enough. Nevertheless, it is a very good baseline for performance measurements.

Here we apply the alternative approach presented in this paper by applying source transformation for repetitive parts of the solver where the operator overloading interconnect the upper level. Numerical results are verified and the cost is compared for most expensive repetitive part of the CFD solver.
5.1 Verification

We verify the sensitivity results for adjoint solver with finite difference for all cells as presented in Equation [3]. The value of step size increment is determined by smallest relative error between the derivatives of adjoint code and finite difference, see [4]. The error comparison in Table 5 show the accuracy of applying dco/fortran and TAPENADE tools for generating derivatives.

$$\text{err} = \sum_{i=1}^{n} |\text{sen}_d(i) - \text{sen}(i)_{\text{version}}|$$

5.2 Test environment

The test case is a S-bend ventilation duct for incompressible steady-state flow as presented in [4]. The objective function computes the loss of pressure at the outlet of the channel and the derived program adds computation of sensitivities of surface nodes with respect to the objective function (pressure loss at outlet). All programs are compiled and run on a machine with hardware specifications shown in Table 6.

<table>
<thead>
<tr>
<th>CPU</th>
<th>Intel(R) Xeon(R) CPU E5-2630</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memory</td>
<td>DDR3 @ 1600 Mhz</td>
</tr>
<tr>
<td>OS</td>
<td>Debian GNU/Linux 8 (jessie)</td>
</tr>
<tr>
<td>Fortan Compiler</td>
<td>GNU Fortran (Debian 4.9.2-10)</td>
</tr>
<tr>
<td>C++ Compiler</td>
<td>gcc (Debian 4.9.2-10) 4.9.2</td>
</tr>
<tr>
<td>S-T tool</td>
<td>TAPENADE 3.9</td>
</tr>
</tbody>
</table>

Table 6: Machine Specifications

5.3 Overloaded program variants

The process described in section [4] aims to combine the overloaded dco/fortran program with kernels derived by TAPENADE. The kernel identification exposed the linear solver as, once again, being the function doing most of the computation. Thus, it is chosen to be replaced using the process presented in sections [4.2.1] and [4.2.2]. Two program variants are compiled which differ in one option for TAPENADE. The option is -nocheckpoint which splits the recording of the variable and control flow stacks from the computation of the adjoint values into two separate functions $f_{\text{fwd}}$ and $f_{\text{bwd}}$ [9]. This allows a more detailed performance comparison. As a baseline for performance the symbolic differentiation of a linear solver is used and incorporated using the external function feature producing a third program variant.
5.4 Performance

Figure 3 and 4 show performance comparison results for the above mentioned test case with 47k, 130k and 500k elements respectively. Runtime and peak memory requirement is measured for computation of sensitivities with a fixed iteration count for the forward computation. The blue solid line is the default variant combining dco/fortran and TAPENADE. Here, the function derived by TAPENADE reruns the forward evaluation of the linear solver during the interpretation of the dco/fortran tape. The black, dashed line represents the second combination where temporary data used by TAPENADE is acquired during the passive execution of the linear solver, i.e. in between steps 1-Forward and 2-Forward. This saves one linear equation solve during the interpretation of each outer iteration but adds to the peak memory requirement. The theoretical maximum memory requirement for this variant is

\[
\text{peak mem} \approx \text{tape} + \sum_{i=1}^{n} \text{stack}_i
\]  

where tape represents the space needed to store the entire dco/fortran tape and stack$_i$ refers to the size of the temporary data of TAPENADE generated for calls of the linear solver during the ith outer iteration. The default variant is between two and three times slower than the symbolic variant for all three mesh configurations. As expected, the -nocheckpoint version (black, dashed line) is faster than the default variant but requires more space. In particular, for smaller test cases (47k) the peak memory depends more on the iteration count of the linear solver as shown in figure 3. Here, the number of iterations needed to meet a certain tolerance of the linear solve about triples after roughly 250 outer iterations. That is the reason for the sudden jump of the peak memory for the variant combining dco/fortran and TAPENADE without checkpoints. A similar but much more pronounced, jump can be observed for the O-O only case. The configuration with 130k elements has a constant high number of iterations of the linear solver and thus does not show a similar jump.
Figure 4: Runtime and Memory for ventilation duct with 130k elements

Figure 5: Runtime and Memory for ventilation duct with 500k elements
6 CONCLUSIONS

The method described in this paper employs a source transformation via operator overloading approach such that the resulting derivative code computes the sensitivity results of the given problem in CFD solver. The key aspect of this method is a combination of source-to-source transformation and operator overloading approaches. The numerical experiments are tested for expensive repetitive part of the solver demonstrating significant memory reduction of the resulting combined code compared to the overloaded adjoint version. In terms of memory usage, these results are competitive with symbolic differentiation of a given function. It could be used as a lower bound to decide whether developing a symbolic adjoint is viable. Moreover, the time needed to compute the derivatives speed up by applying the combined approach which is clearly superior for higher number of iteration. The overall design of a generic transformation methodology is presented that is applicable for complicated computer programs.

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