

## IMPLEMENTATION OF POD AND DMD METHODS IN APACHE SPARK FRAMEWORK FOR SIMULATION OF UNSTEADY TURBULENT FLOW IN THE MODEL COMBUSTOR

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**Abstract.** *The paper is devoted to modelling and analysis of unsteady turbulent flow in a model combustor (channel) using LES (Large Eddy Simulation). Simulations were provided for 2D and 3D cases on different grids of a flow in a channel with rearward facing step. The calculation of a flow in a channel was performed on high performance cluster, using the new approach in Apache Spark framework and POD, DMD data processing algorithms. First 4 dynamic modes were defined.*

## 1 INTRODUCTION

Solution of Continuum Mechanics problems (hydrodynamics, turbulent combustion, aerodynamics and aeroacoustics) is an important direction for solving fundamental and applied tasks. Usually, numerical simulations are carried out with detailed grids (tens and hundreds of millions of cells) using high performance clusters. Scale turbulence resolution requirements impose certain restrictions on the cells size when using Hybrid RANS/LES and LES methods. As a result, calculation of unsteady flow generates large amounts of data (tens of terabytes), processing of which requires considerable time and computing resources.

## 2 PROBLEM STATEMENT

This paper considers the problem of incompressible subsonic flow simulation in a channel with a rearward facing step. This kind of flows occurs in various technical devices, including combustion chambers of aircraft engines and helicopters. The main feature of such flows is the formation of large-scale structures in the mixing layer behind a step, which is used as a flame stabilizer [1,2]. The extension of the mixing layer is determined by the dynamics of large vortices development, which are formed upstream and increase as the result of neighboring vortices merging and inclusions of viscous gases from the main flow (Fig.1). The mixing process inside of the large scale structures is determined by small-scale turbulence. The Reynolds number in our case is 22000.

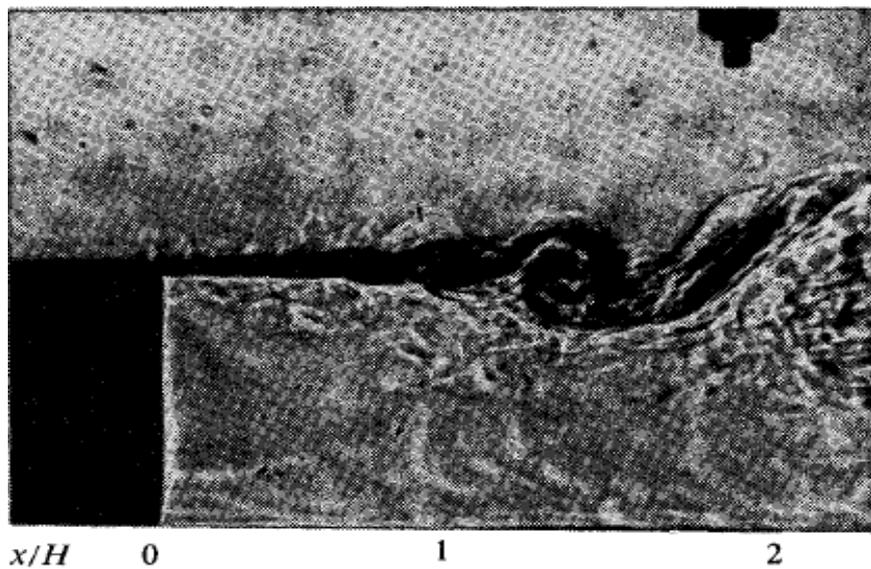


Figure 1: Schlieren photograph.

## 3 MATHEMATIC MODEL AND CALCULATION RESULTS

In this paper, the authors used open source package OpenFOAM, pisoFoam solver [3], for calculation of unsteady incompressible flow in a channel with a rearward facing step (Fig.2).

For the simulation of unsteady flow it is recommended to use Large Eddy Simulation (LES) model. Large-scale vortex structures are calculated by integrating the filtered Navier-Stokes equations [2,3]. To obtain the filtered equations the boxed filter is used. Small vortices, sized not exceeding the pitch of the computational grid, are simulated using the Smagorinsky model and model based on one differential equation for sub-grid kinetic energy. Finite volume method is used for discretization of the equations. The calculation time step is  $dt = 10^{-5}$ . The velocity at entrance were set at  $U = 10$  m/s taking into account the overlapping of perturba-

tions for the simulation of a turbulent flow. The nonpermeability conditions were set on the walls. Continuation of the flow boundary condition was set at the exit. Calculation scheme has the second order of accuracy. The grid is based on hexahedrons. The block grid with 13 blocks was built-up for 2D and 3D calculations (Fig.3). The number of cells for 2D case ranged from 626 up to 2690 (Fig.4). For 3D case there were 244500 cells.

The resulting equation for the relation of velocity and pressure are solved by the iterative PISO algorithm. The system of linear equations was solved by the method of conjugate gradients with preconditioner.

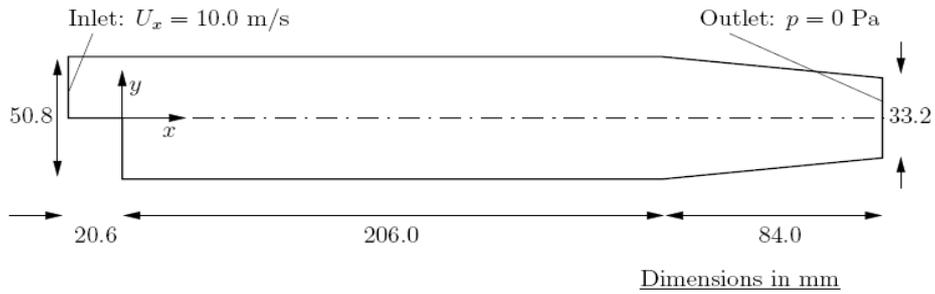


Figure 2: Domain of computation Characteristic dimensions.

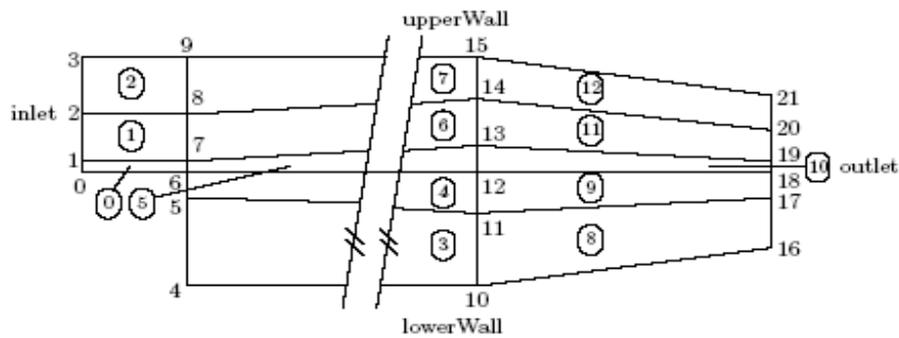


Figure 3: Domain of computation and blocks.

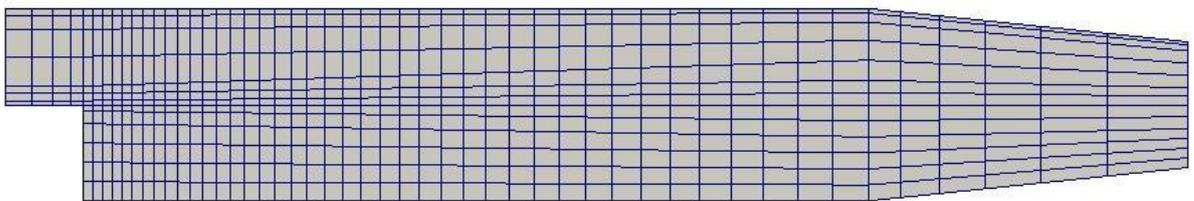


Figure 4: Coarse grid

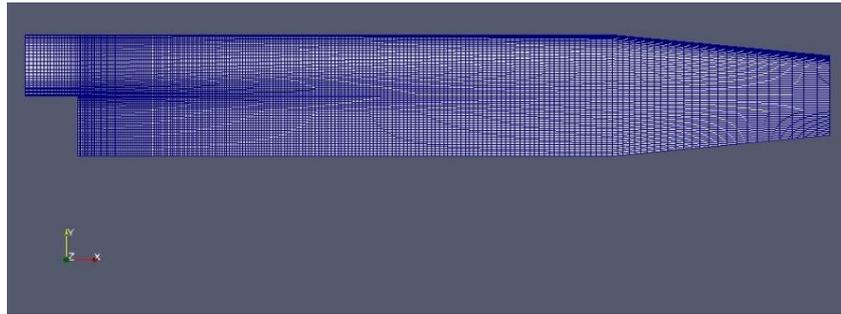


Figure 5: Detailed (Fine) grid.

The calculation results analysis by Paraview of the velocity magnitude value had revealed that the large-scale coherent structures, 3D vortex are formed behind a rearward facing step. Animated picture of the flow around the step has a pronounced unsteady flow character. Periodic self-quenching of the large-scale vortices were registered with a frequency of  $f=200$  Hz. Figures 6-9 illustrate the results of the velocity magnitude calculation. The simulation cases were run with 24 cores on high performance cluster of ISP RAS.

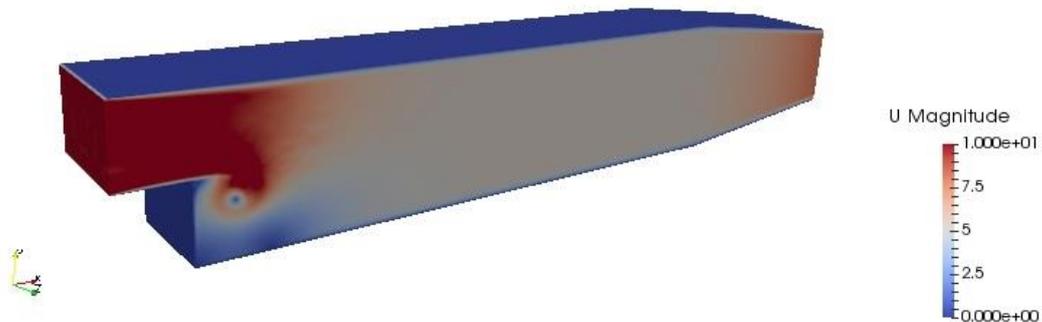


Figure 1: Field of velocity magnitude at the time  $t=0.005$  s.

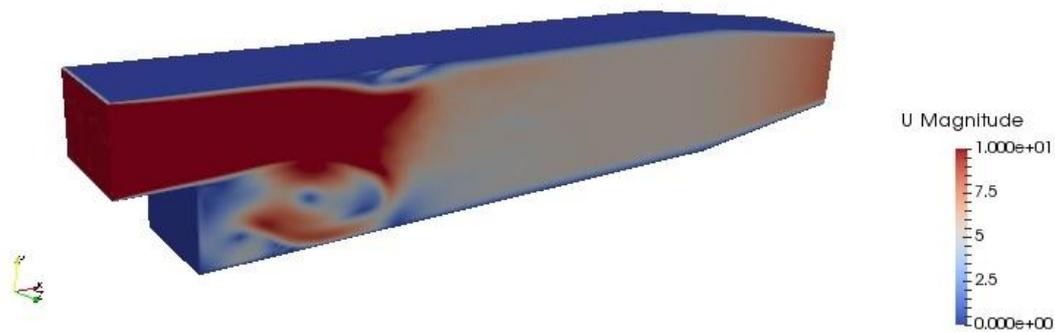
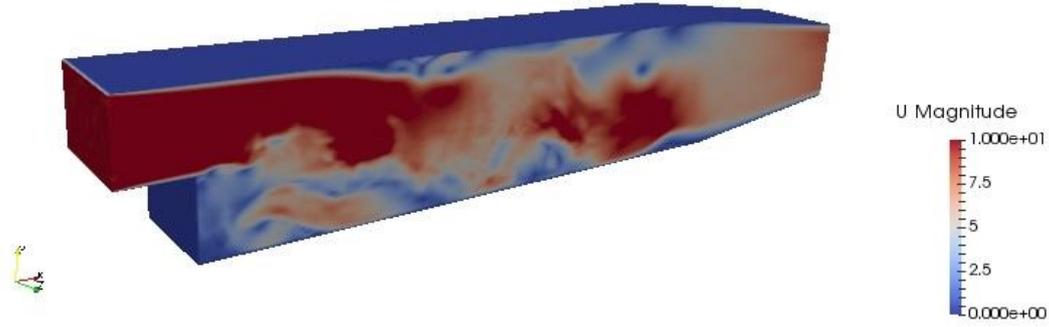
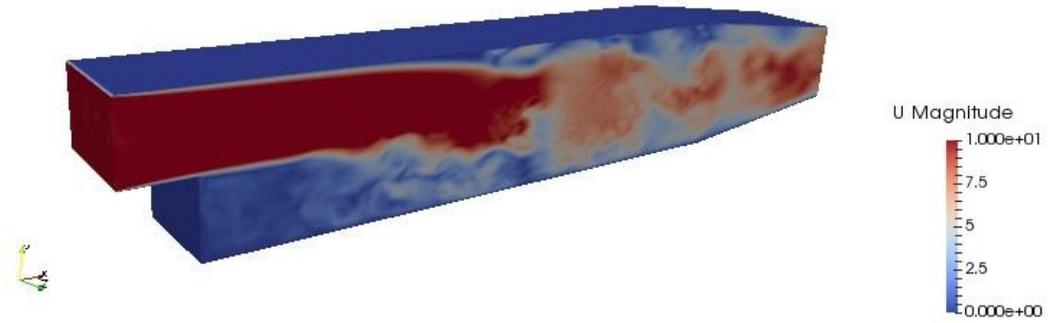


Figure 7: Field of velocity magnitude at the time  $t=0.02$  s.


 Figure 8: Field of velocity magnitude at the time  $t=0.05$  s.

 Figure 9: Field of velocity magnitude at the time  $t=0.1$  s.

#### 4 APACHE SPARK TECHNOLOGY AND POD AND DMD METHODS

Apache Spark technology allows to process large amounts of data in ram memory by distributing the load between the compute nodes, ensuring high fault tolerance (Fig. 10). When calculating the flow process, considerable amount of data in the form of separate time snapshots of the flow was obtained.

The POD and DMD algorithms allow to obtain new data on turbulence and coherent structures. Authors of this paper have applied POD and DMD algorithms using Apache Spark framework and LAPACK mathematical library.

The POD was first introduced by Lumley in the field of Computational Fluid Dynamics [4]. The present day analysis uses method of snapshots introduced by Sirovich [5]. To compute the POD using the Eq. 1 requires solving  $n \times n$  eigenvalue problem.

$$\int_0^T R(t_1, t_2) \phi(t_2) dt_2 = \lambda \phi(t_1) \quad (1)$$

Where  $R(t_1, t_2)$  is the auto-covariance of the fluid variable.

The main problem is the calculation of auto-covariance matrix  $R(t_1, t_2)$ . Snapshots method proposes that auto-covariance matrix  $R(t_1, t_2)$  can be approximated by a summation of snapshots.

$$R_{ij} = \frac{1}{M} \sum_{n=1}^M x_i^n x_j^n \quad (2)$$

The snapshots are assumed to be distanced by a time or a special distance greater than the correlation time or distance.

The input data for DMD [6-8] have to be presented in the form of sequence of snapshots and are set by a matrix  $V$  with the size  $n \times m$ , where

$$V_1 = [v_1, v_2, v_3, \dots, v_{n-1}] \quad (3)$$

A linear mapping from one snapshot to another is assumed

$$V_1^N = \{v_1, Av_1, \dots, A^{N-1}v_1\} \quad (4)$$

By the linear combination of available data fields, we have a standard Arnoldi iteration problem.

$$Av^{n-1} \approx v^{n-1}S \quad (5)$$

Utility written in C++, allowed transferring the data obtained during the hydrodynamic calculation from the OpenFOAM format into the library format, written in Scala.

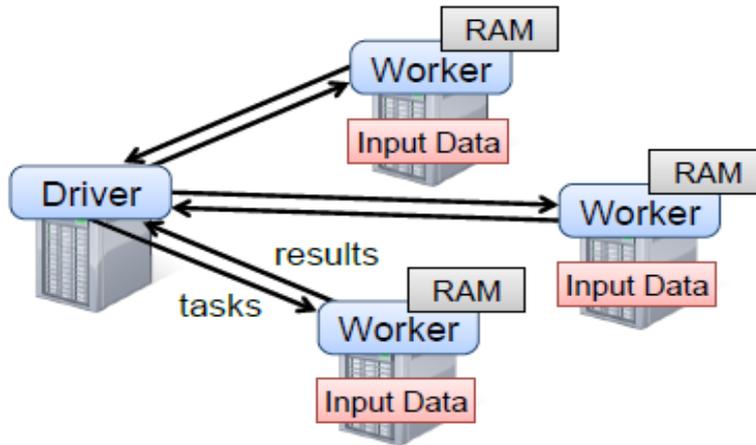


Figure 10: Architecture of Apache Spark technology.

To highlight the characteristic structures, authors propose to use POD (Proper Orthogonal Decomposition) method. For the data processing, 26 snapshots and calculation results of velocity magnitude field were used. As a result of POD and DMD algorithms implementation, the basic dynamic modes, characterizing the large-scale structure in the channel have been revealed. 4 most characteristic modes were selected (Fig.11-14). The arrows on figures display the velocity field taken at  $t = 0.25$  s. Contribution of the other modes to the total turbulence energy is insignificant.

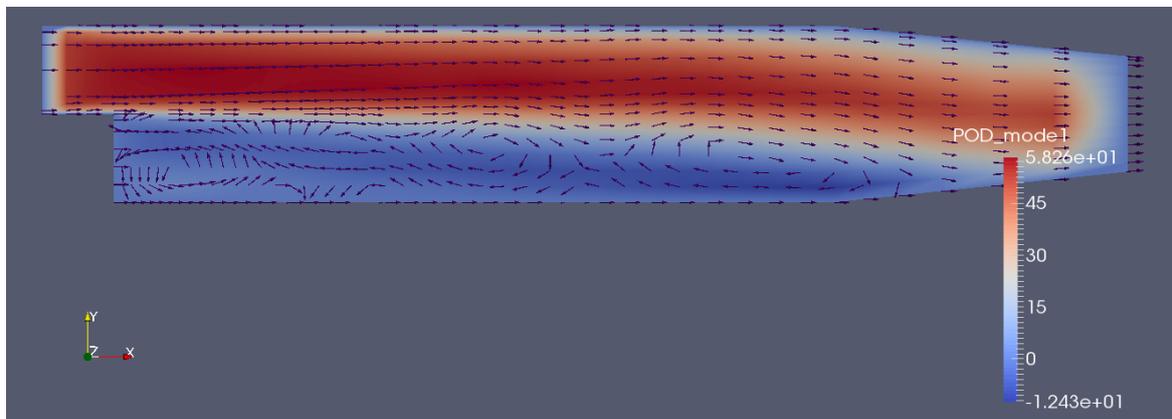


Figure 11: 1-st POD mode.

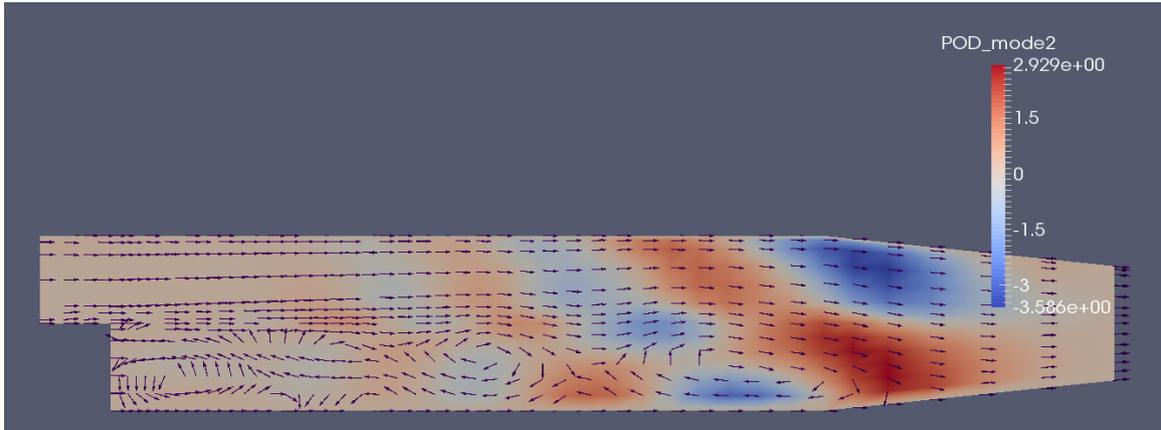


Figure 12: 2-nd POD mode.

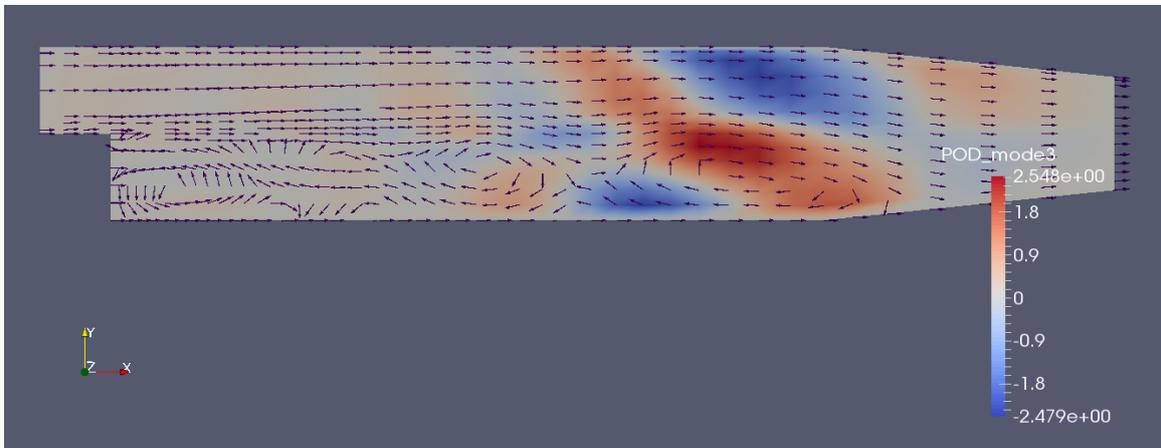


Figure 13: 3-rd POD mode.

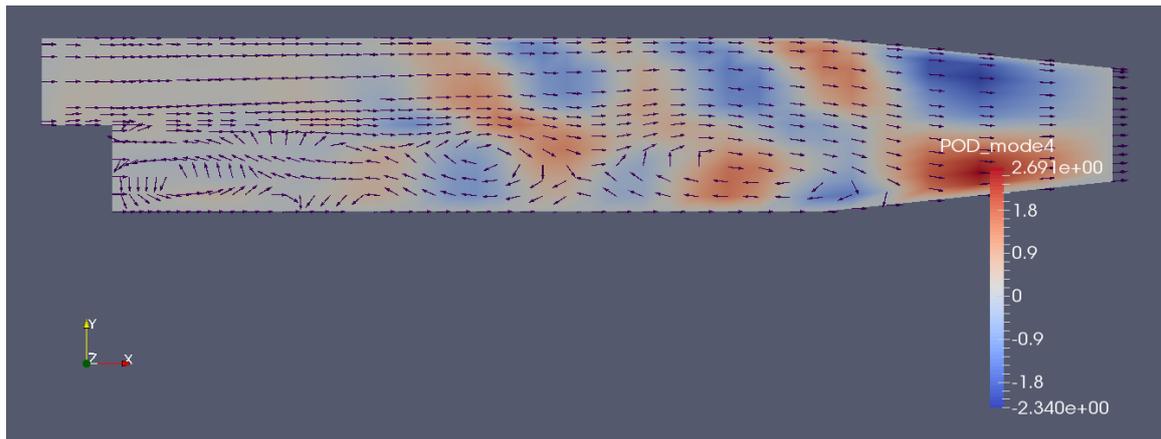


Figure 14: 4-th POD mode.

As a rule, the different modes correspond to the position of different vortices in channel. The four most significant singular values are shown in table 1. This values capture 87% of the kinetic energy.

Number of value	Absolute value	% of captured energy
1	24,14653	80,23%
2	0,897377	83,21%
3	0,716828	85,59%
4	0,690438	87,89%

Table 1: First 4 modes.

## 5 CONCLUSIONS

The calculation of the flow in the channel was performed, using the new approach in Apache Spark and POD, DMD algorithms, first 4 dynamic modes are selected. Application of this technology allows to solve more complex problems for investigation of turbulence flows and obtaining data on characteristics of coherent structures.

## ACKNOWLEDGMENTS

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## REFERENCES

- [1] R.W. Pitz, J.W. Daily. Experimental study of combustion in a turbulent free shear layer formed at a rearward facing step. *AIAA Journal*, **21**, 1565-1570, 1983.
- [2] C. Fureby. Homogenization Based LES for Turbulent Combustion. *Flow, Turbulence and Combustion*, **84**, 459-480, 2010.
- [3] H.G. Weller, G. Tabor, H. Jasak, C. Fureby. A tensorial approach to computational continuum mechanics using object oriented techniques, *Computers in Physics*, **12**, № 6, 620-631, 1998.
- [4] G. Berkooz, P. Holmes, and J. Lumley. The proper orthogonal decomposition in the analysis of turbulent flows, *Annual review of fluid mechanics*, **25**, no. 1, pp. 539–575, 1993.
- [5] L. Sirovich. Turbulence and the Dynamics of Coherent Structures: Parts I, II, and III. *Quarterly of Applied Mathematics*, **45**, 561–590, 1987.
- [6] P.J. Schmid. Dynamic mode decomposition of numerical and experimental data. *Journal of Fluid Mechanics*, **656**, 5-28, 2010.
- [7] P. J. Schmid, L. Li, M. P. Juniper, O. Pust. Applications of the dynamic mode decomposition. *Theor. Comp. Fluid Dyn.*, **25**, 249-259, 2011.
- [8] C. Rowley, I. Mezic, S. Bagheri, P. Schlatter, D. Henningson. Spectral analysis of non-linear flows. *Journal of Fluid Mechanics*, **641**, 115–127, 2009.