On the verification of high-order CFD solvers

Farshad Navah*, Siva Nadarajah

McGill University
Computational Aerodynamics Group, Department of Mechanical Engineering
Montreal, QC H3A 2S6 Canada
farshad.navah@mail.mcgill.ca, siva.nadarajah@mcgill.ca

ABSTRACT

Most current state-of-the-art industrial computational fluid dynamics (CFD) solvers are 2nd order accurate at best. This class of solvers, called low-order, are prone to large discretization errors that compromise the use of CFD as a reliable method for the estimation of engineering quantities such as lift and drag coefficients. High-order accurate methods have the potential to produce lower discretization errors for the same number of degrees of freedom as their low-order counterparts and hence have been the focus of an increasing interest from the research community. However, the higher accuracy of implemented schemes is not always verified and demonstrated in research publications due to the lack of sufficiently complex analytical solutions for flow regimes of interest. Instead, most authors directly proceed to validation by comparing flow and output quantities to experimental data.

Verification is a mathematical exercise to determine if the numerical solution at hand is that of the discrete set of governing equations; hence, it answers to the question: ”are the equations solved correctly?” On the other hand, Validation is rather concerned with the ability of the mathematical model to represent the physical phenomenon with fidelity. In this sense, validation answers to the question: ”are the right equations solved?”

One avenue to extend the code verification to more significant flow regimes is via the method of manufactured solutions (MMS) [1]. In this approach, a flow solution is arbitrarily defined with the aim of testing the entirety or part of the discretization of the governing equations and corresponding boundary conditions. The manufactured solution is then used to define forcing functions and boundary quantities that serve to solve a modified version of the original discrete equations. The difference between the solution of the modified equations and the manufactured solution yields a quantifiable exact error, that is a function of the order of accuracy of the numerical scheme.

This paper studies the application of the MMS to the verification of Euler, Navier-Stokes and Reynolds-averaged Navier-Stokes equations, discretized by a high-order correction procedure via reconstruction (CPR) scheme [2].

The manufactured solutions for each equation along with the targeted flow regime are presented. The important aspects of the implementation, the true error levels and the resulting orders of accuracy are discussed. Finally, recommendations on the verification of high-order discretizations of flow problems are proposed.

References

[1] Roache, P. J., Verification and Validation in Computational Science and Engineering, *Hermosa Publishers*, 1998
[2] Cagnone, J., Vermeire, B., and Nadarajah, S., A p-adaptive {LCP} formulation for the compressible Navier-Stokes equations, *Journal of Computational Physics*, Vol. 233, No. 0, 2013, pp. 324 – 338.